Square Roots

Objective 1 Understand the meaning of a Square Root

What does it mean to **square** a number?
If we square 8, we have $8 \cdot 8 = 8^2 = 64$.

If we **square root** 64, we get 8. The mathematical symbol for square root is $\sqrt{}$. We also call it a radical.

> Note: It is note the long division symbol $\overline{)}$.

The mathematically statement $\sqrt{64}$ is asking us "what is the square root of 64". In other words, "what positive number do you square to get 64".

There are actually two integers you can square to get 64. These are -8 and 8. But the square root function only gives the "principal root". Which means the square root of a number is always positive.

Finally, we can make the statement

$$\sqrt{64} = 8.$$

The square of an integer is known as a perfect square. There are several perfect squares you should already know. These are 144, 121, 100, 81, 64, 49, 36, 25, 16, 9, 4, 1, 0.

Notice the following.

$\sqrt{144} = 12$ $\sqrt{25} = 5$

$\sqrt{121} = 11$ $\sqrt{16} = 4$

$\sqrt{100} = 10$ $\sqrt{9} = 3$

$\sqrt{81} = 9$ $\sqrt{4} = 2$

$\sqrt{64} = 8$ $\sqrt{1} = 1$

$\sqrt{49} = 7$ $\sqrt{0} = 0$

$\sqrt{36} = 6$

Most square roots we need a calculator to calculate like for $\sqrt{2}$. We can only approximate it answer. For example, $\sqrt{2} = 1.414$ rounded to the nearest one-thousandth. We will only be working with square roots of perfect squares in this section.

Objective 2

Evaluate Expressions with Square Roots

Step 1 of the rules for order of operations states to "Perform all the operations within a **parenthesis** or other **grouping symbols**". The square root symbol, or radical, is considered a grouping symbol. Therefore, if there is an expression under a square root, you must first simplify the expression beneath the radical symbol before taking the square root.

Example 1: Evaluate each expression following the rules for Order of Operations.

a) $\sqrt{20-4^2}$

$\sqrt{20-16}$

$\sqrt{4}$

$\boxed{2}$

c) $3\sqrt{16}$

$3\cdot\sqrt{16}$

$3\cdot 4$

$\boxed{12}$

e) $-2\sqrt{36}-\left(\sqrt{25}-3\right)$

b) $\sqrt{6^2+13}$

d) $5\sqrt{25}$

f) $\left(\sqrt{1}-\sqrt{121}\right)^2-2\sqrt{81}$

Answer the following homework questions.

In Exercises 1 - 12, evaluate each expression following the rules for order of operations.

1) $\sqrt{1}+\sqrt{0}$

2) $\sqrt{144}-2\sqrt{36}$

3) $-\sqrt{49}+|-6|$

4) $\left|\sqrt{81}-\sqrt{100}\right|$

5) $3\sqrt{4}$

6) $\sqrt{4}+\sqrt{4}+\sqrt{4}$

7) $\sqrt{81}\div\sqrt{9}$

8) $\sqrt{(36\div 6)^2}$

9) $\sqrt{-2^2+20}$

10) $\sqrt{61-(3-8)^2}$

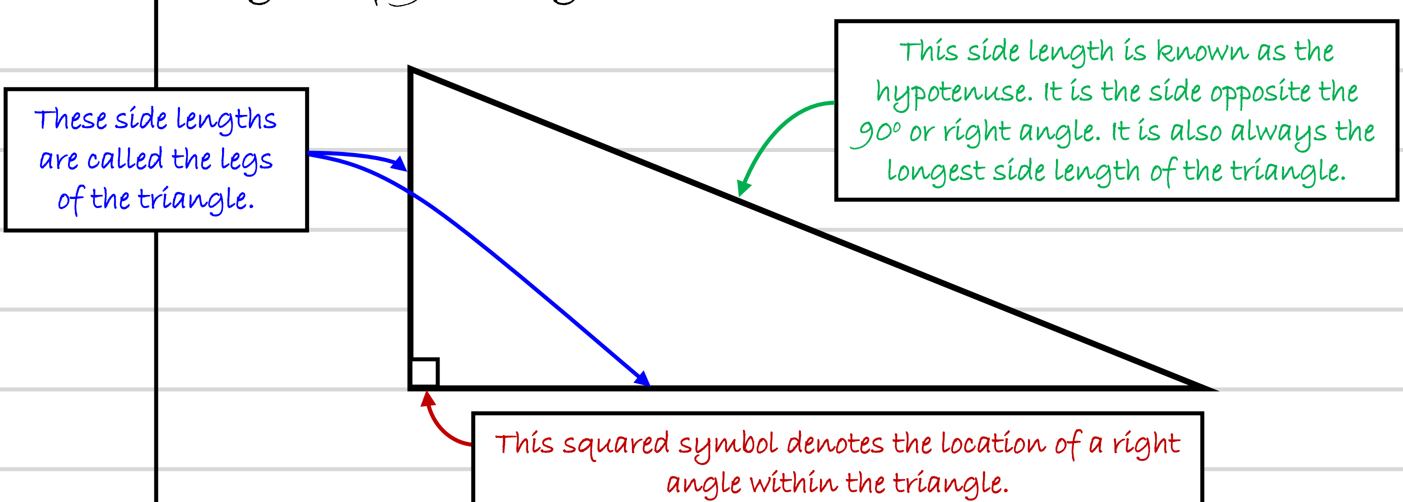
11) $\sqrt{\frac{1}{4}}$

12) $\sqrt{\frac{64}{16}}$

Objective 3

Understand the Pythagorean Theorem for Right Triangles

A **right triangle** is a triangle that has a right angle of 90 degrees (90°).



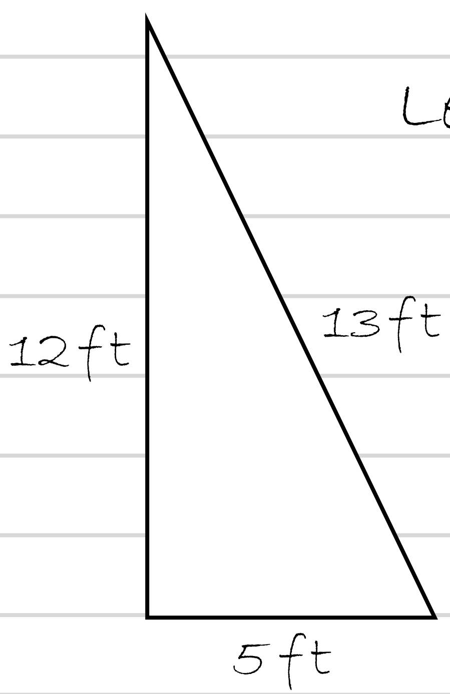
The Pythagorean Theorem states,

$$a^2+b^2=c^2$$

where a and b represent the lengths of the legs of the triangle (in no particular order) and c represents the length of the hypotenuse.

The side lengths of any right triangle must satisfy the theorem. If the side lengths do not, then it is not a right triangle!

Example 2: Show that the triangle below is a right triangle using the Pythagorean Theorem.



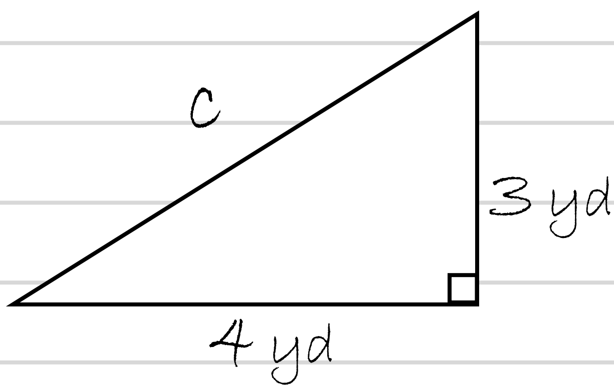
Let $a=5$, $b=12$, and $c=13$.

$$a^2+b^2=c^2$$
$$5^2+12^2=13^2$$
$$25+144=169$$
$$169=169\ \checkmark$$

The Pythagorean Theorem is satisfied, therefore this is a right triangle. The right angle is located opposite the hypotenuse which is 13 ft in length.

Example 3: Find the length of the hypotenuse in the triangle.

Let $a=3$ and $b=4$.



$$a^2+b^2=c^2$$
$$3^2+4^2=c^2$$
$$9+16=c^2$$
$$25=c^2$$

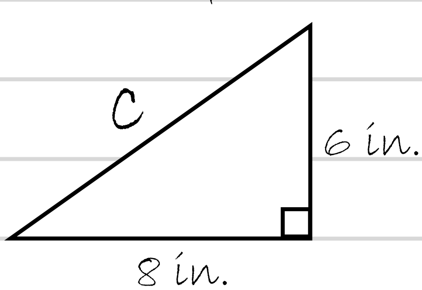
Since $25=c^2$ or $c^2=25$, we as ourselves "what number do we square to get 25?" Since the square root of 25 is 5 ($\sqrt{25}=5$), this means that c must equal 5.

Therefore the length of the hypotenuse in the triangle above must be 5 yards.

Answer the following homework question.

In Exercise 13-14, find the missing side length in each right triangle.

13)



14)