Reducing Fractions

Objective 1 | Write a Fraction in Lowest Terms (Reducing)

> Note: A fraction is written in **lowest terms** or **reduced** when the numerator and denominator have no common factors other than 1.

Let's begin with the fraction $\frac{3}{8}$. In this case, both the numerator **3** and denominator **8** have no common factors other than 1. Therefore, this fraction is in lowest terms.

Now let's look at $\frac{6}{8}$. Here, the numerator and denominator have a common factor of 2. To reduce this fraction we divide out the common terms between the numerator and denominator. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

Notice that dividing both the numerator and denominator by the same factor results in an equivalent fraction!

In some cases, we may have to divide out common factors more than once to reduce the fraction to lowest terms.

Let's take a look at the fraction $\frac{28}{42}$. It may not be obvious that 28 and 42 have a common factor of 14. Since they are both even numbers we can begin by dividing out the common factor of 2.

$$\frac{28}{42} = \frac{28 \div 2}{42 \div 2} = \frac{14}{21}$$

Since 14 and 21 both have a common factor of 7, the fraction $\frac{14}{21}$ is not written in lowest terms. Therefore we divide out the common factor of 7.

$$\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$$

Since 2 and 3 have no common factors other than 1, we can now state that $\frac{28}{42}$ is now written in lowest terms as the equivalent fraction $\frac{2}{3}$.

Since we separately divided out a common factor of **2** and **7**, this means that $2 \cdot 7$ or **14** is a common factor of 28 and 42. Therefore, the original fraction $\frac{28}{42}$ could have been reduced in one step using the common factor of 14.

$$\frac{28}{42} = \frac{28 \div 14}{42 \div 14} = \frac{2}{3}$$

Recall that $-21 \div 3 = -7$. When writing this quotient using a fraction bar, we have $\frac{-21}{3}$. Dividing out the common factor of 7, we get the following result.

$$\frac{-21}{3} = \frac{-21 \div 3}{3 \div 3} = \frac{-7}{1} = -7$$

Suppose we were given $21 \div (-3)$ which also equals -7. Using a fraction bar we have the equation $\frac{21}{-3} = -7$.

We can see that $\frac{-21}{3} = \frac{21}{-3} = -7$. Therefore, whenever we have one negative sign in either the numerator or denominator (not both), we can move it to the front of the fraction to indicate a negative answer.

$$\frac{-21}{3} = \frac{21}{-3} = -\frac{21}{3} = -7$$

Answer the following homework questions.

In Exercises 1 – 6, write each fraction in lowest terms.

1) $\frac{6}{20}$ 3) $\frac{6}{51}$ 5) $\frac{42}{28}$

2) $\frac{8}{52}$ 4) $\frac{6}{54}$ 6) $\frac{80}{24}$

Let's now reduce a fraction without writing a division symbol. Suppose we are asked to reduce $\frac{8}{10}$. Since 8 and 10 are both divisible by 2, we will divide out this common factor using the following notation.

$$\frac{8}{10} = \frac{\cancel{8}^{4}}{\cancel{10}_{5}} = \frac{4}{5}$$

Here it is understood that you are dividing out a common factor of 2.

Below is the same process without writing down a division symbol.

$$\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

Objective 2 ## Reducing Fractions that have Variables

Let's begin with the fraction $\frac{8x^3}{x^2}$. Remember that exponents are used to represent repeated multiplication. Therefore $\frac{8x^3}{x^2} = \frac{8 \cdot x \cdot x \cdot x}{x \cdot x}$.

Since $\frac{x}{x} = 1$, we can cancel variable terms as follows.

$$\frac{8x^3}{x^2} = \frac{8 \cdot \cancel{x}^{1} \cdot \cancel{x}^{1} \cdot x}{\cancel{x}_{1} \cdot \cancel{x}_{1}} = \frac{8x}{1} = 8x$$

Notice that when cancelling variable terms, they get replaced with **1**'s.

Suppose we were given $\dfrac{20n^2}{6n^5}$. Again remember that exponents are used to represent repeated multiplication. Therefore we use the following notation to reduce the fraction to lowest terms.

$$\dfrac{20n^2}{6n^5} = \dfrac{\overset{10}{\cancel{20}} \cdot \overset{1}{\cancel{n}} \cdot \overset{1}{\cancel{n}}}{\underset{3}{\cancel{6}} \cdot \underset{1}{\cancel{n}} \cdot \underset{1}{\cancel{n}} \cdot n \cdot n \cdot n} = \dfrac{10}{3 \cdot n \cdot n \cdot n} = \dfrac{10}{3n^3}$$

Answer the following homework questions.

In Exercises 7 – 15, write each fraction in lowest terms.

7) $\dfrac{15}{20}$ 10) $\dfrac{3a}{3b}$ 13) $\dfrac{2xy}{8xyz}$

8) $\dfrac{80}{24}$ 11) $\dfrac{4xy}{-4y}$ 14) $\dfrac{3a^2b^3}{ab}$

9) $\dfrac{12}{52}$ 12) $\dfrac{5a}{-5ab}$ 15) $\dfrac{7a^2b^3c^5}{63ab^4c^7}$