CK-12 FlexBook

Early Math - World Possible

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Contents

CHAPTER **1** # CK-12 Algebra Explorations Concepts, Pre-Kindergarten

Chapter Outline

Introduction

In these concepts, you will be introduced to the basic concepts of algebra and will practice your problem solving skills. There are five concepts, and each one focuses on a key algebraic thinking strategy. You will focus on describing, solving, and checking your thinking.

1.1 Picture Graphs

Students look at picture graphs in order to count and compare numbers of objects.

What do you see in the picture graph below? Can you say who has more or less? In this Concept, we will practice looking at picture graphs and describing what we see by counting and using comparison words like **more** and **less**.
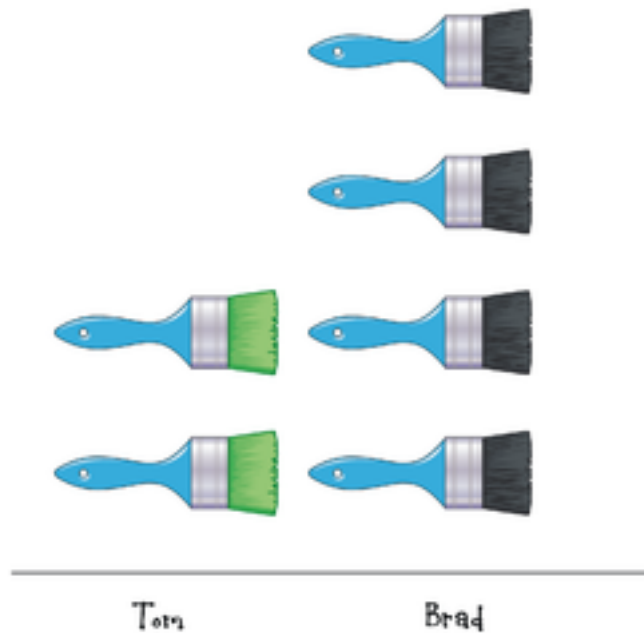
Guidance

When looking at a picture graph like the one above we see different stacks of objects. In order to describe what we see we can say what object it is. We can also count how many objects are in each stack. We can also compare the stacks and say which one has more or less.

Example A

Look at this picture graph:
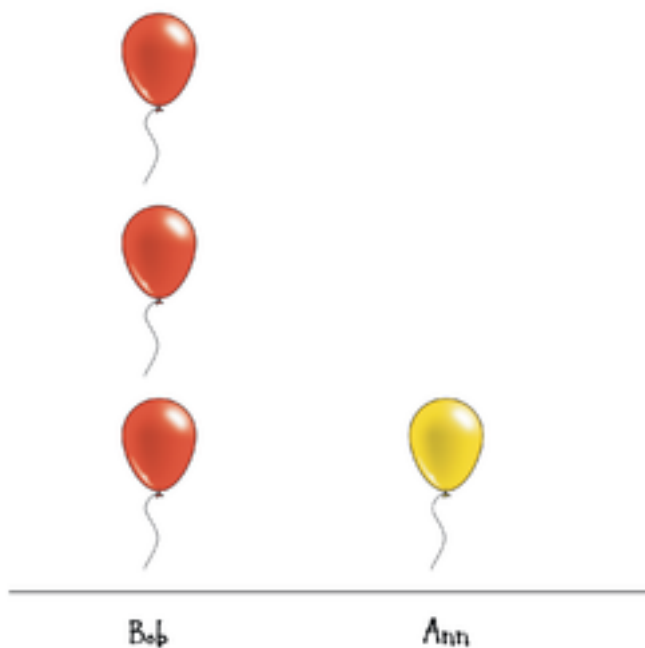
What do you see?

You should see some red balloons and some yellow balloons. The red balloons belong to Bob. It says Bob under the stack of red balloons. The yellow balloons are Ann's. It says Ann under the stack of yellow balloons.

Who has more balloons?

You should see that there are more red balloons. Bob has more balloons because Bob has the red balloons.

Who has less balloons?

You should see that there are less yellow balloons. Ann has less balloons because Ann has the yellow balloons.

How many red balloons does Bob have?

Try counting starting from the top balloon. Put your finger on each balloon as you count. You should count 3 red balloons.

How many yellow balloons does Ann have?

Try counting starting from the top balloon. Put your finger on each balloon as you count. You should count 1 yellow balloon.

Example B

Look at this picture graph:
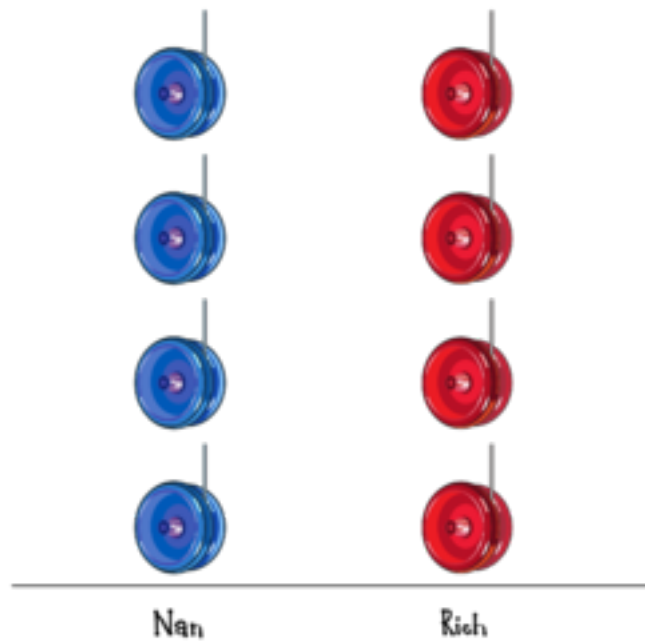
What do you see?

You should see two stacks of yoyos. One stack has red yoyos. The other stack has blue yoyos. The red yoyos belong to Rich. The blue yoyos belong to Nan.

How many red yoyos belong to Rich?

Try counting starting from the top yoyo. Put your finger on each yoyo as you count. You should count 4 red yoyos.

How many blue yoyos belong to Nan?

Try counting starting from the top yoyo. Put your finger on each yoyo as you count. You should count 4 blue yoyos.

Rich and Nan have the same number of yoyos. How can you tell by looking at the stacks?

You should notice that the two stacks are the same height. That's why Rich and Nan have the same number of yoyos.

Concept Problem Revisited
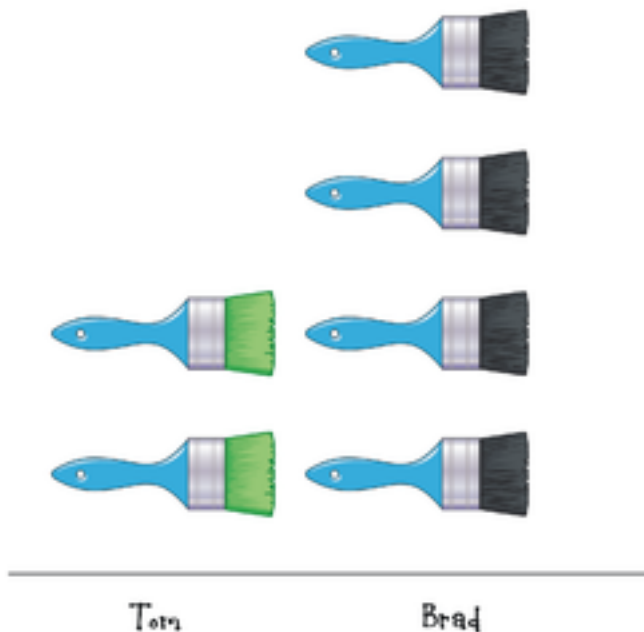
In this picture you should see two stacks of paint brushes. One stack has green paint brushes. The other stack has black paint brushes. Brad has 4 black paint brushes and Tom has 2 green paint brushes. Brad has more paint brushes. Tom has less paint brushes.

Vocabulary

To have *more* means to have a greater amount. To have *less* means to have a smaller amount. In the picture below, there are *more* red balloons. There are *less* yellow balloons.
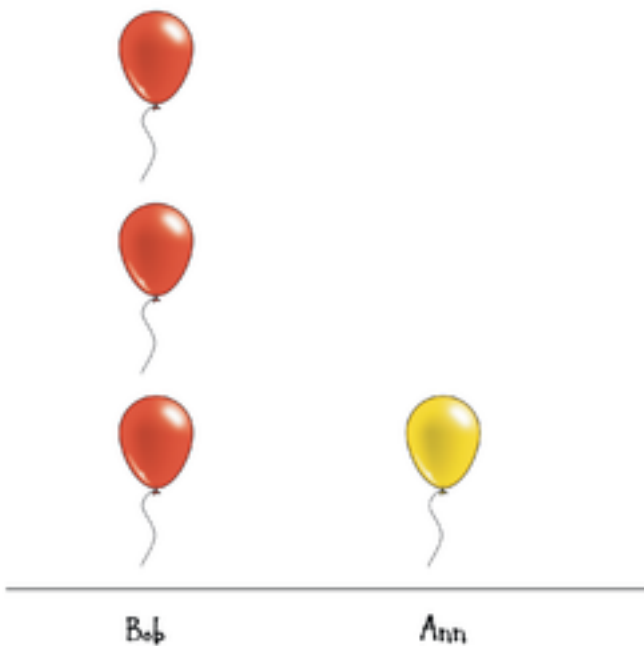
Guided Practice

1. What do you see? Who has the most crayons? Who has the least crayons? How many crayons does each person have?
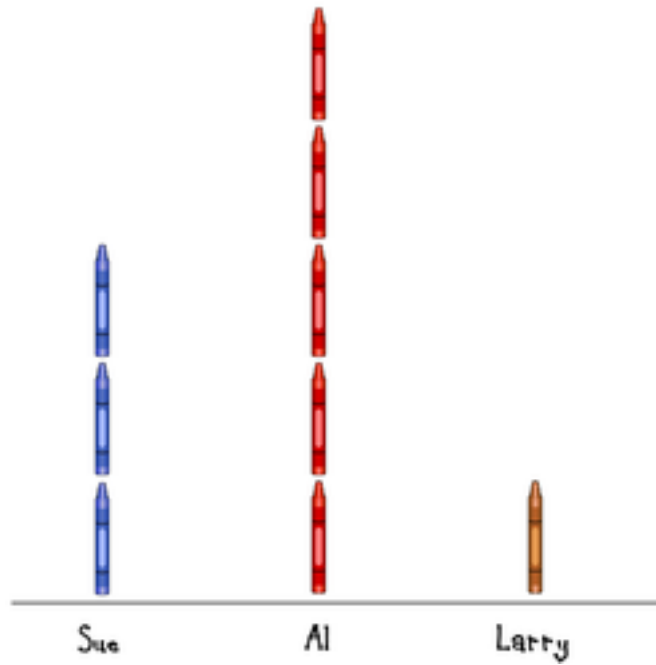
2. What do you see? Who has the most beach balls? How many beach balls does each person have? Do any stacks have the same number of beach balls? How do you know?

3. What do you see? Who has the most puzzles? Who has the least puzzles? How many puzzles does each person have?

Greg   Eric   Sandy

Answers:

1. There are three stacks of crayons. One stack has purple crayons. One stack has red crayons. One stack has brown crayons. Al has the most crayons. He has 5 red crayons. Larry has the least crayons. Larry has 1 brown crayon. Sue has 3 purple crayons.

2. There are three stacks of beach balls. One stack has pink beach balls. One stack has blue beach balls. One stack has yellow beach balls. Nina has the most beach balls. Nina has 5 yellow beach balls. Ron 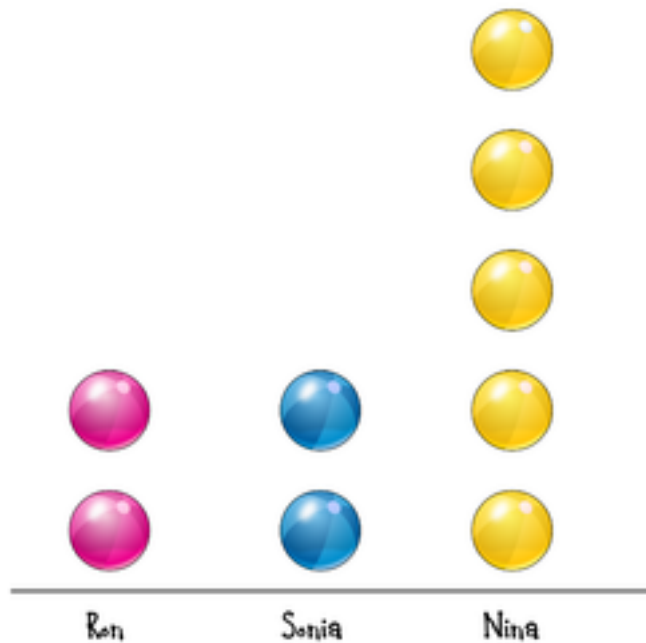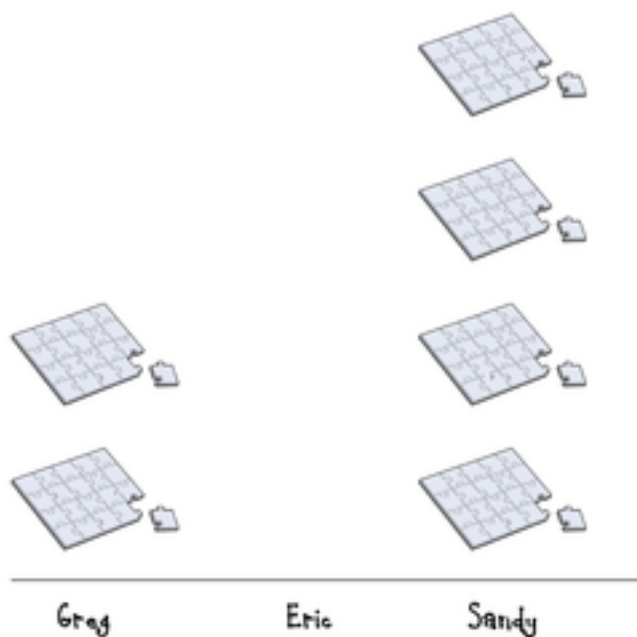and Sonia have the same number of beach balls. Ron has pink beach balls and Sonia has blue beach balls. They each have 2 beach balls. Their stacks are the same height.

3. There are three names, but only two stacks of puzzles. Sandy has the most puzzles. She has 4 puzzles. Eric has the least puzzles, he has 0 puzzles. The other stack of puzzles belongs to Greg. Greg has 2 puzzles.

Explore More

For problems 1, 2, and 3: Look at the picture. What do you see? Who has the most? Who has the least? Do any people have the same number?

1.

2.

3.
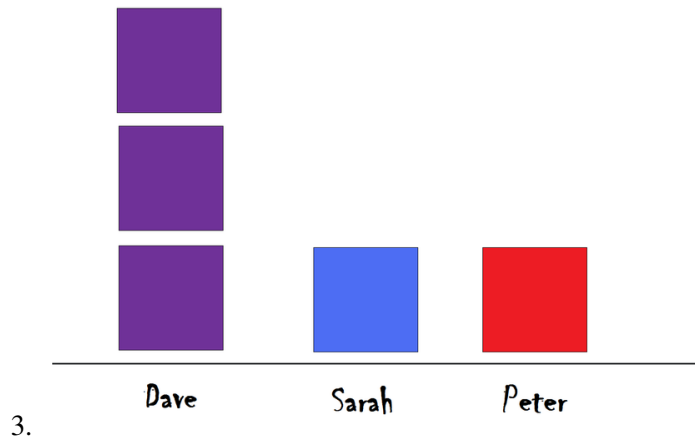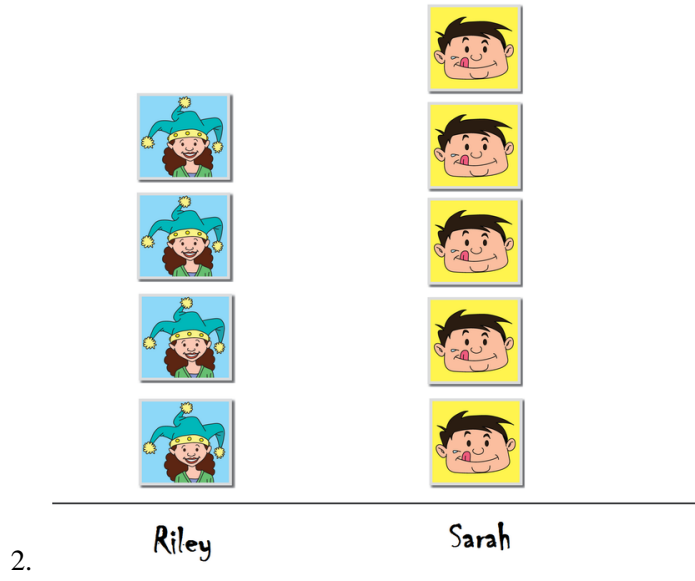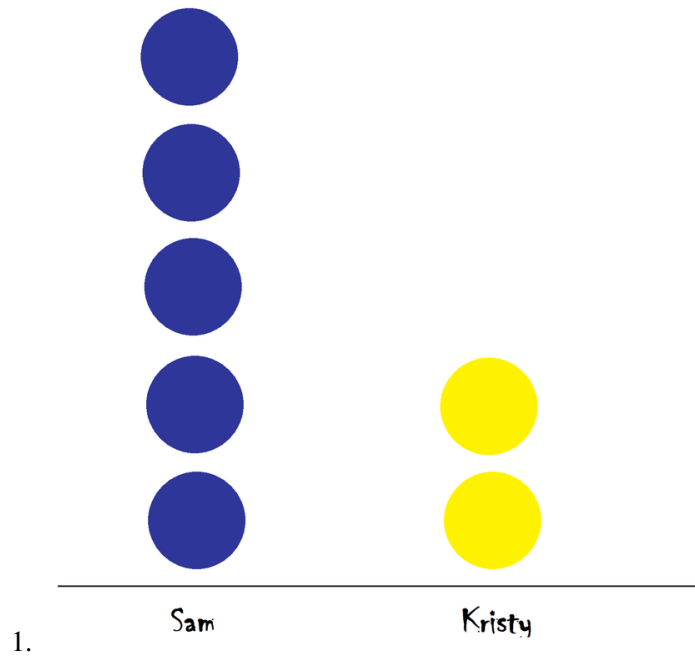
4. Mike and Lisa have squares. Mike has red squares and Lisa has blue squares. Mike has more squares than Lisa. Draw a picture graph that matches this description.

5. Mark, Samantha, and Rose have stuffed animals. Mark has less stuffed animals than Rose. Samantha has the

same number of stuffed animals as Rose. Draw a picture graph that matches this description.

1.2 Train Cars

Students look for patterns in pictures of train cars in order to fill in missing information.

Do you notice a pattern in the train cars below? Can you fill in the missing cars? In this Concept, we will practice describing the patterns we see in pictures and completing the patterns.

Guidance

When looking at a picture like the picture of the train cars above, sometimes we can look for a pattern. When looking for a pattern, read the colors aloud and see if you notice any set of colors happening more than once. After you figure out the pattern, you can fill in the blank cars so that they match the pattern.

Example A

Look at the train cars below and describe what you see. Do you notice a pattern?

You should notice that the colors Red, Blue, Yellow repeat three times. There is also a Red car at the end before the caboose.

What colors should the blank train cars be?

In order to fit with the pattern, the blank train cars should be Red, Blue, and then Yellow. Check that the pattern sounds right by naming all the colors of the cars aloud.

Example B

Look at the train cars below and describe what you see. Do you notice a pattern?

You should notice that the colors Red, Yellow, Yellow repeat three times. There is also a Red car at the end after the last two Yellow cars.

What colors should the blank train cars be?

In order to fit with the pattern, the blank train cars should be Yellow, Yellow, and then Red. Check that the pattern sounds right by naming all the colors of the cars aloud.

Concept Problem Revisited

In this set of train cars, the colors Blue, Red, Red, Yellow are repeated. In order to fit with the pattern, the three blank cars should be Yellow, Red, Red.

Vocabulary

One type of *pattern* is when certain characteristics (like color) get repeated many times in the same order. The train cars below have a pattern in their colors. The colors Red, Blue, and then Yellow repeat three times.

Guided Practice

For each set of train cars below, describe the pattern. Then, fill in the colors for the blank cars. Finally, read the colors of all the cars aloud to check that your solution sounds right.

1.

2.

3.

Answers:

1. In this set of train cars, the colors Yellow, Yellow, Blue, Blue are repeated. In order to fit with the pattern, the blank cars should be Yellow, Yellow, Blue.

2. In this set of train cars, the colors Red, Blue, Blue, Yellow are repeated. In order to fit with the pattern, the blank cars should be Red, Blue, Blue.

3. In this set of train cars, the colors Red, Red, Yellow, Blue are repeated. In order to fit with the pattern, the blank cars should be Red, Yellow, Red, Red, Yellow.

Explore More

For each set of train cars below, describe the pattern. Then, fill in the colors for the blank cars. Finally, read the colors of all the cars aloud to check that your solution sounds right.

1.

2.

3.

4.

5.

1.3 Teeter Tottering

Students look at teeter totters in order to describe what they see and compare the weights of different objects.

Can you tell which animal is heavier in the picture below? Can you tell which animal is lighter? In this Concept, we will practice describing what we see and identifying what is heavier and what is lighter.

Guidance

When looking at a picture of a teeter totter like the picture above, we will often see that one side is on the ground and one side is not. This tells us which object on the teeter totter is heavier and which object is lighter. The side of the teeter totter that is on the ground will have the heavier object on it. The side of the teeter totter that is not on the ground will have the lighter object on it.

Example A

Look at the picture below and describe what you see:
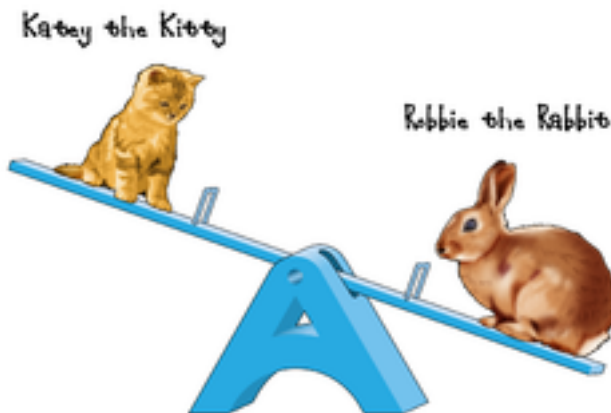
You should notice two animals on a teeter totter. One rabbit and one kitty. The kitty's name is Katey. The rabbit's name is Robbie.

Who is heavier?

You should notice that Robbie is heavier.

How can you tell from the picture?

You can tell because the rabbit's side is lower because he weighs more.

Example B

Look at the picture below and describe what you see:
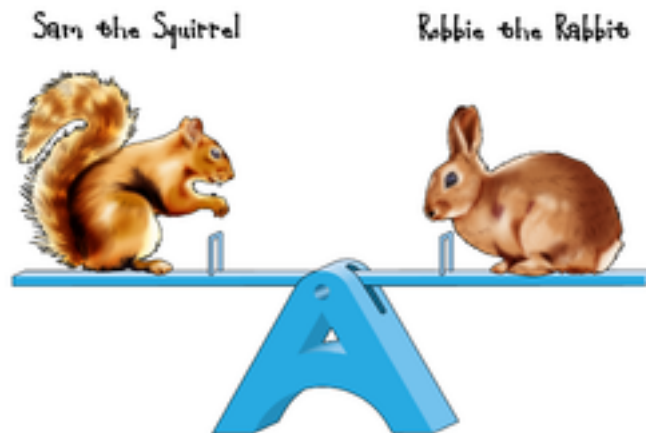
You should notice two animals on a teeter totter. One is a squirrel and one is a rabbit. The squirrel is named Sam. The rabbit is named Robbie.

These two animals weigh the same. How can you tell from the picture?

You can tell because the teeter totter is level. Neither of the sides are lower than the other.

Concept Problem Revisited
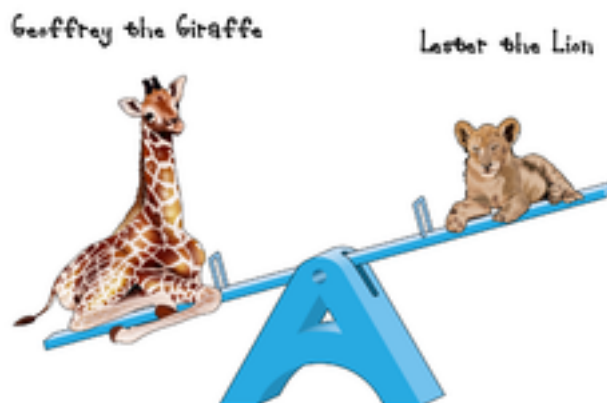
In this picture we see a teeter totter with Geoffrey the giraffe and Lester the lion. We can see that Geoffrey is heavier. The giraffe's side is lower because he weighs more. We can see that Lester is lighter. The lion's side is higher because he weighs less.

Vocabulary

One object is *heavier* than another object if it weighs more. One object is *lighter* than another object if it weighs less. We can compare the weights of objects by looking at them on a teeter totter.

Guided Practice

For each teeter totter below, describe what you see. Then, identify which animal is heavier, which animal is lighter, or if the two animals weigh the same amount. Explain how you can tell from the picture.
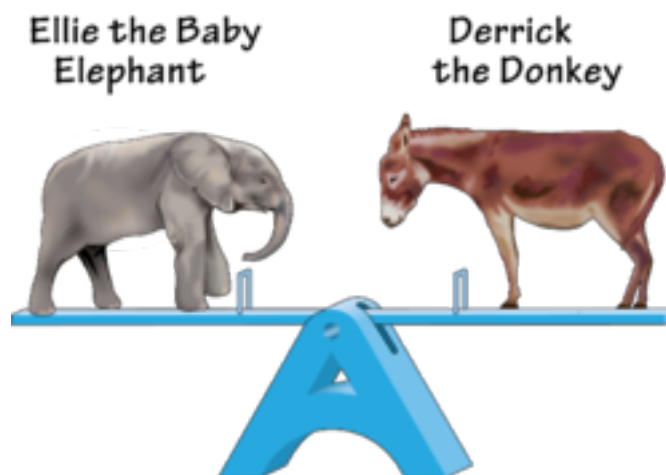
1.

Orly the Otter

Cassie the Camel

2.

Ellie the Baby
Elephant

Derrick
the Donkey

3.

Harry the Hippopotamus   Andrea the Aardvark

Answers:

1. You should see 2 animals on a teeter totter: an otter and a camel. The otter is named Orly and the camel is named Cassie. Cassie is heavier because the camel is at the bottom of the teeter totter. Orly is lighter because the otter is at the top of the teeter totter.

2. You should see 2 animals on a teeter totter: a baby elephant and a donkey. The elephant is named Ellie and the donkey is named Derrick. Ellie and Derrick weigh the same. We can tell because the teeter totter is level.

3. You should see 2 animals on a teeter totter: a hippopotamus and an aardvark. The hippopotamus is named Harry and the aardvark is named Andrea. Andrea is lighter. We can tell because the aardvark is at the top of the teeter totter. Harry is heavier. We can tell because the hippopotamus is at the bottom of the teeter totter.

Explore More

For each teeter totter below, describe what you see. Then, identify which animal is heavier, which animal is lighter, or if the two animals weigh the same amount. Explain how you can tell from the picture.

Geoffrey the Giraffe   Andrea the Aardvark
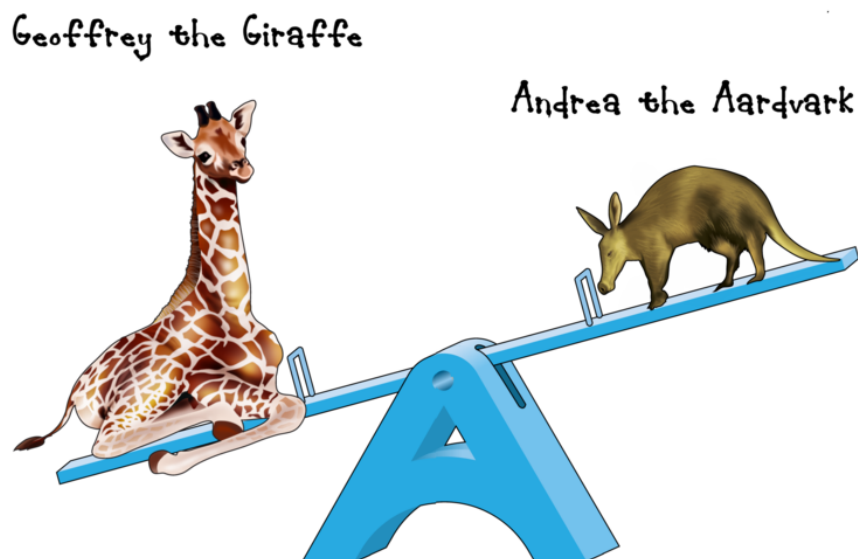
1.

Harry the Hippopotamus   Lester the Lion

2.

IMAGE NOT AVAILABLE
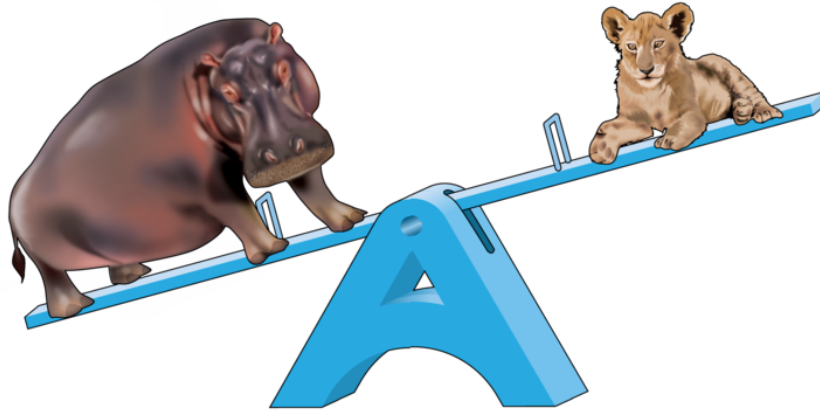
3.

Orly the Otter   Katey the Kitty

4.

5. Draw your own picture of a teeter totter that shows two animals weighing the same amount.
6. Draw your own picture of a teeter totter that shows one animal weighing more than another animal. Circle the
    animal that is heavier.

1.4 Packing Bags

Students identify shapes in a filled bag, fill empty bags with the same shapes as were in the filled bag, and then count the total numbers of shapes.
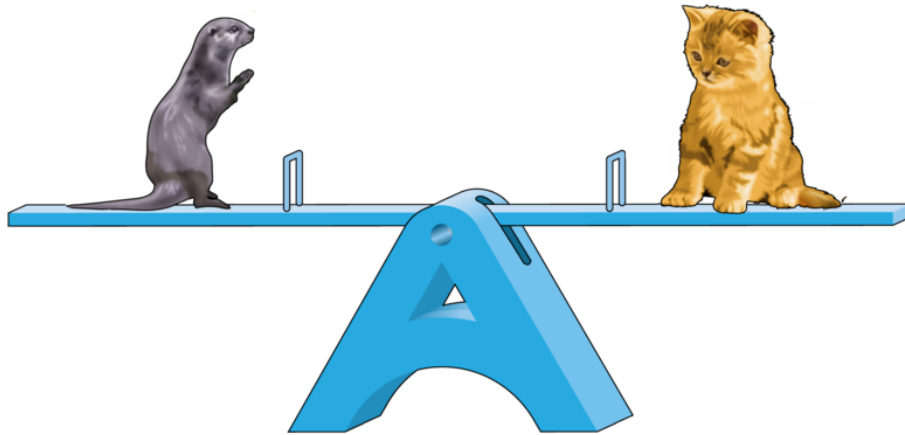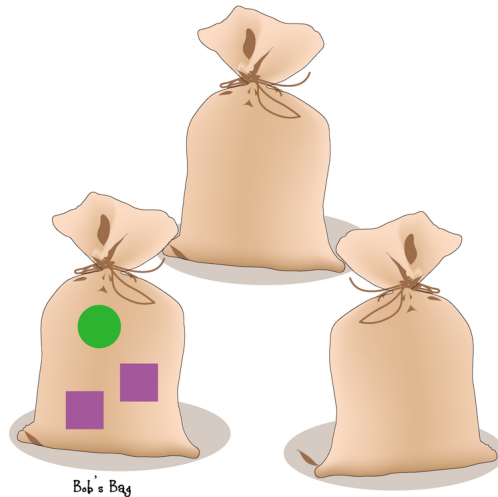
Can you describe what you see in the picture below? What if you put the same shapes that are in the filled bag into the empty bags? How many of each shape would you now have? In this Concept, we will practice describing what we see and counting shapes.

Bob's Bag

Guidance

When looking at bags like in the picture above, we can see that one bag is filled and the other bags are empty. We can fill the empty bags the same way as the filled bag. Then, we can count to see how many of each shape we have in total.

Example A

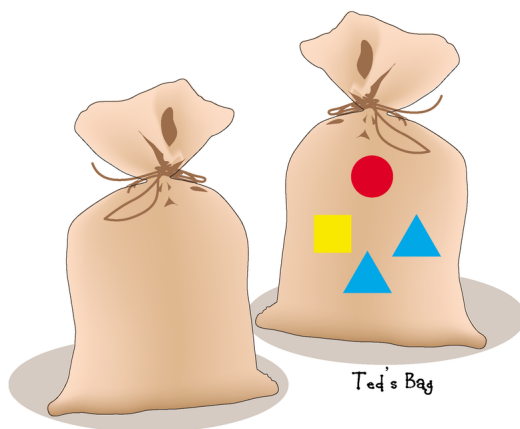*Look at the picture below. What do you see?*

Ann's Bag

You should notice 2 bags. One bag has shapes in it and the other bag is empty. The bag with the shapes in it has 2 circles and 1 triangle.

The bag with shapes in it is Ann's bag. Use your shape tiles. Put 1 triangle into the empty bag. Put 2 circles into the empty bag. Now, count the shapes. How many circles in all? How many triangles in all?

You should have 4 circles in all and 2 triangles in all.

Example B

Look at the picture below. What do you see?

Ted's Bag

You should notice 2 bags. One bag has shapes in it and the other bag is empty. The bag with the shapes in it has 2 triangles, 1 circle, and 1 square.

The bag with the shapes in it is Ted's bag. Use your shape tiles. Put 2 triangles into the empty bag. Put 1 circle into the empty bag. Put 1 square into the empty bag. Count the shapes. How many triangles in all? How many circles in all? How many squares in all?

You should have 4 triangles in all, 2 circles in all, and 2 squares in all.

Concept Problem Revisited
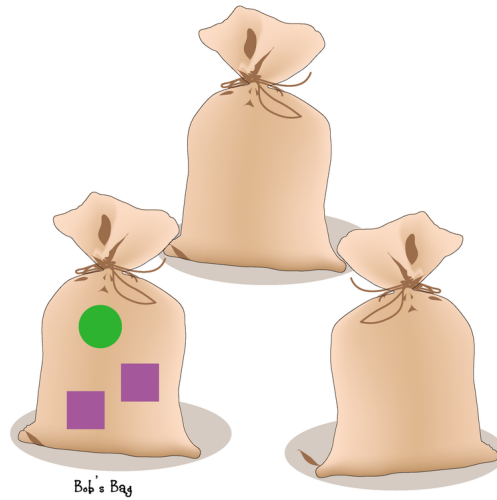
Bob's Bag

You should notice 3 bags. One bag is Bob's bag and it has 1 circle and 2 squares in it. The other two bags are empty. If you put 2 squares into each empty bag and 1 circle into each empty bag you will have 6 squares in all and 3 circles in all.

Vocabulary

This is a *circle:*
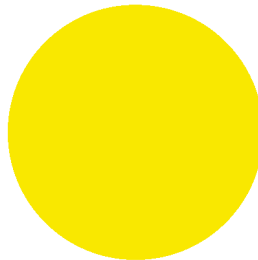
This is a *square:*

This is a *triangle:*

Guided Practice

For each set of bags below, describe what you see. Then, fill the empty bag(s) with the same shapes that are in the filled bag. Last, count how many of each shape you have in all.

1.

2.

3.

Debbie's Bag
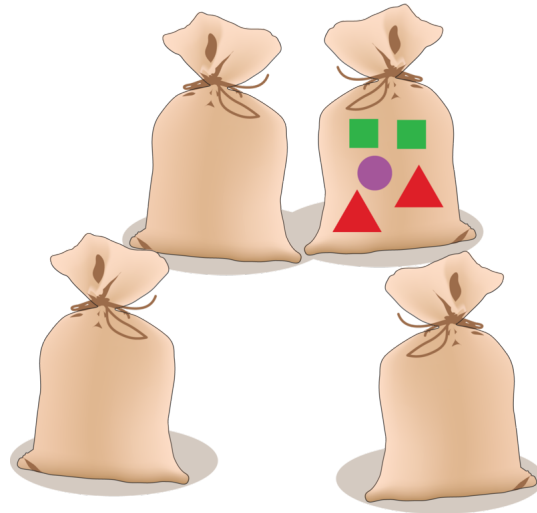
Answers:
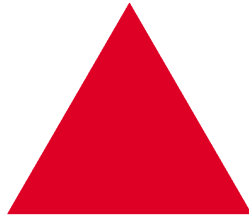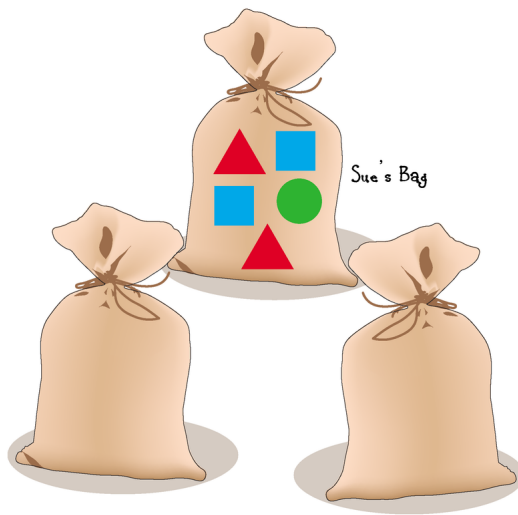
1. You should notice 3 bags. One bag has 2 square, 2 triangles, and 1 circle. The other 2 bags are empty. If you put 2 squares, 2 triangles, and 1 circle in each empty bag, you will have 6 squares in all, 6 triangles in all, and 3 circles in all.

2. You should notice 3 bags. One bag has 3 circles, 1 triangle, and 1 square. The other 2 bags are empty. If you put 3 circles, 1 triangle, and 1 square into each empty bag, you will have 9 circles in all, 3 triangles in all, and 3 squares in all.

3. You should notice 4 bags. One bag has 2 triangles, 1 circle, and 2 squares. The other 3 bags are empty. If you put 2 triangles, 1 circle, and 2 squares into each empty bag, you will have 8 triangles in all, 4 circles in all, and 8 squares in all.

Explore More

For each set of bags below, describe what you see. Then, fill the empty bag(s) with the same shapes that are in the filled bag. Last, count how many of each shape you have in all.

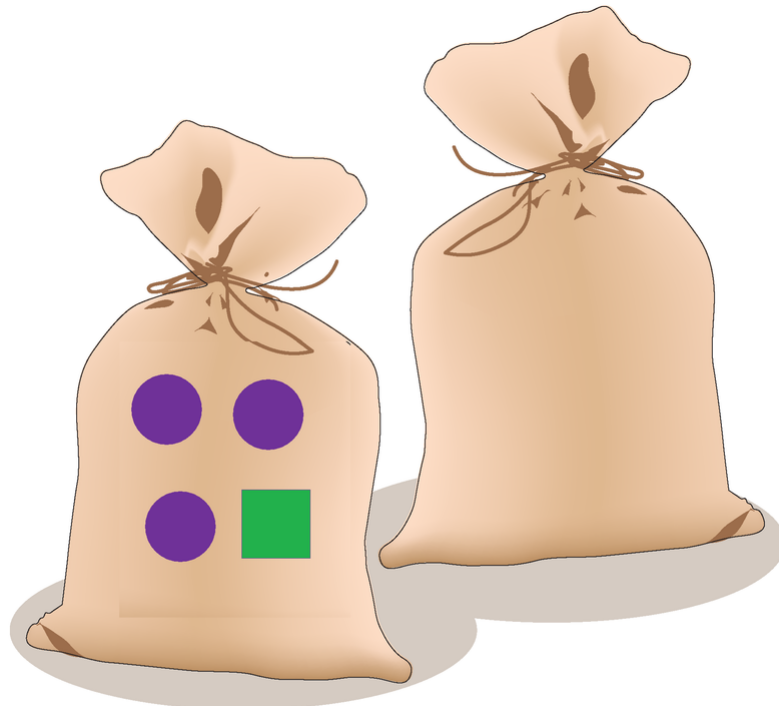Ted's Bag

1.

Rochelle's Bag

2.

Dan's Bag

3.

Justin's Bag

4.

5. What if there had been two empty bags in #1 instead of just one empty bag? If you filled all of the empty bags, how many of each shape would there be?
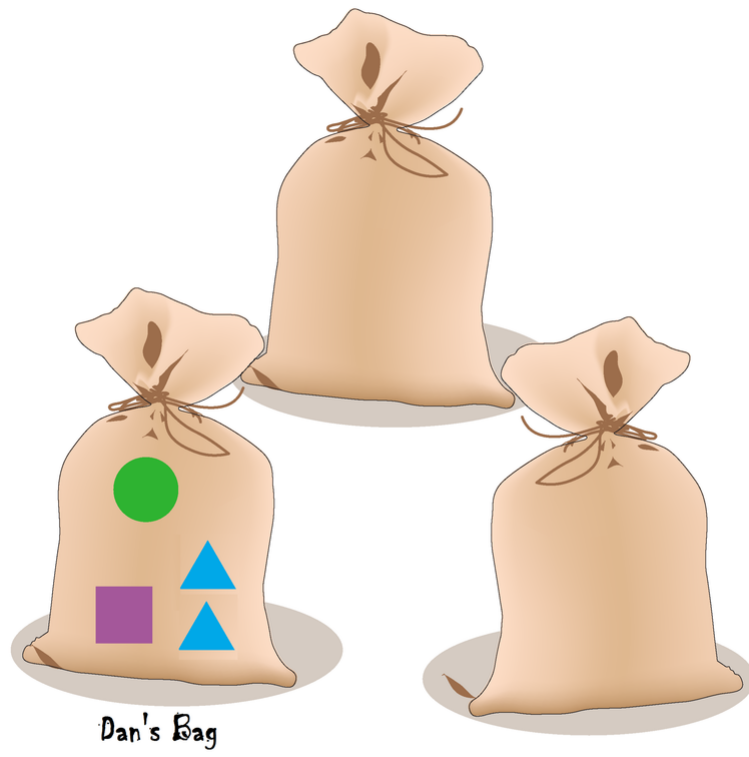6. What if there had been three empty bags in #1 instead of just one empty bag? If you filled all of the empty bags, how many of each shape would there be?
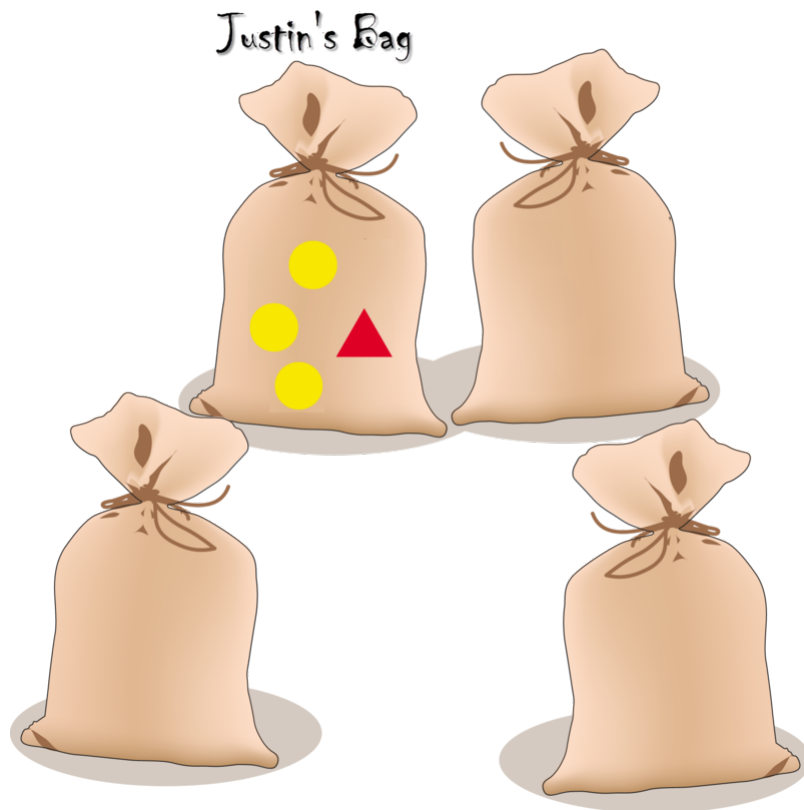7. What if there had been four empty bags in #1 instead of just one empty bag? If you filled all of the empty bags, how many of each shape would there be?

1.5 Using Clues

Students look at pictures and clues to identify the character being described.

Can you match a description or a clue with a picture? If you know that Mr. Rippledandy is one of the pictures below and Mr. Rippledandy has a hat, can you say who Mr. Rippledandy might be? In this Concept, we will practice using clues to identify characters.
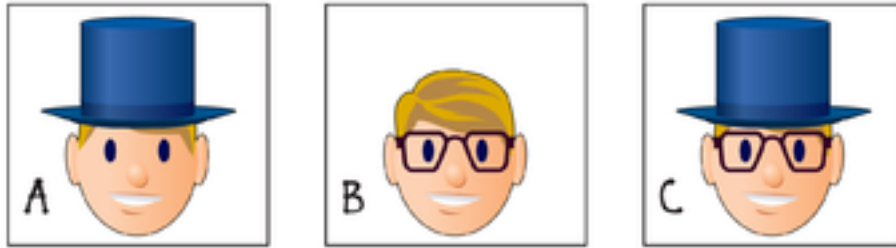
Guidance

When looking at a set of similar pictures like the faces above, we need clues to figure out which picture is correct. For each clue, see which pictures match the clue. The correct picture will be the one that matches all of the clues.

Example A

Look at the pictures below. Peter Palooka's pumpkin has a triangle nose. Which pumpkins match this clue?

You should notice that pumpkins B and C both have a triangle nose. We know that Peter Palooka's pumpkin is not pumpkin A.

Peter's pumpkin has a mouth. Which pumpkin is Peter Palooka's pumpkin?

Pumpkin B must be Peter Palooka's pumpkin because pumpkin B is the only pumpkin with both a triangle nose and a mouth.

Example B

Look at the pictures below. Mr. Liketysplitity's Umbrella has 2 circles. Which umbrellas might be Mr. Liketysplitity's?
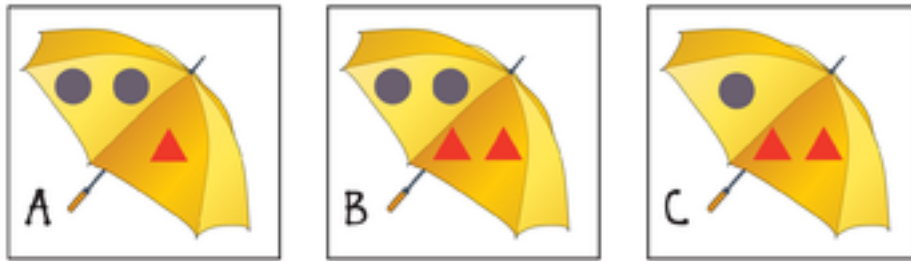
You should notice that umbrellas A and B have 2 circles. We know that Mr. Liketysplitity's umbrella is not umbrella C.

Mr. Liketysplitity's umbrella has one triangle. Which umbrella is Mr. Liketysplitity's umbrella?

Umbrella A must be Mr. Liketysplitity's umbrella because umbrella A is the only umbrella with both 2 circles and one triangle.

Concept Problem Revisited
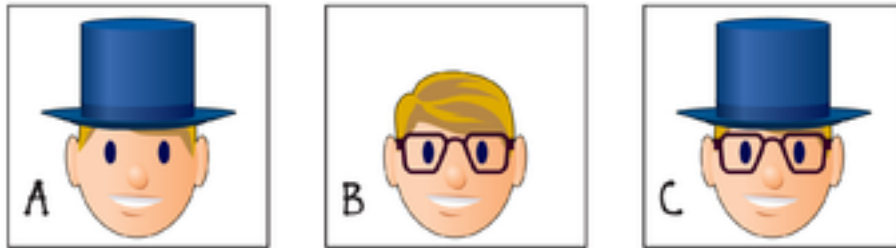
Because Mr. Rippledandy has a hat, he must be Face A or Face C. If we also know that Mr. Rippledandy is wearing eyeglasses then we know that Mr. Rippledandy must be Face C.

Vocabulary

A *clue* is a piece of information that helps us to solve a puzzle or mystery. When we use clues, we are using *deductive reasoning*.

Guided Practice

1. Mr. Bobbity's Book has 1 snake on the cover. His book has 3 spiders on the cover. Which book is Mr. Bobbity's book?
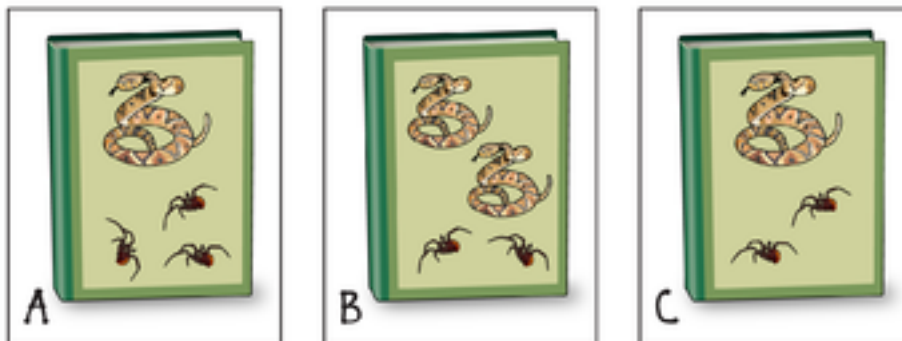
2. Anna's Balloon has one dot. Her balloon has 4 stars. Which balloon is Anna's balloon?
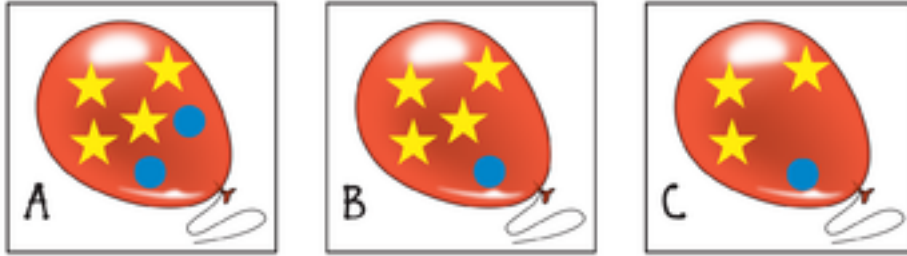
3. Mrs. Starglitter's Purse has 2 stripes. Her purse has 5 squares. Which purse is Mrs. Starglitter's purse?
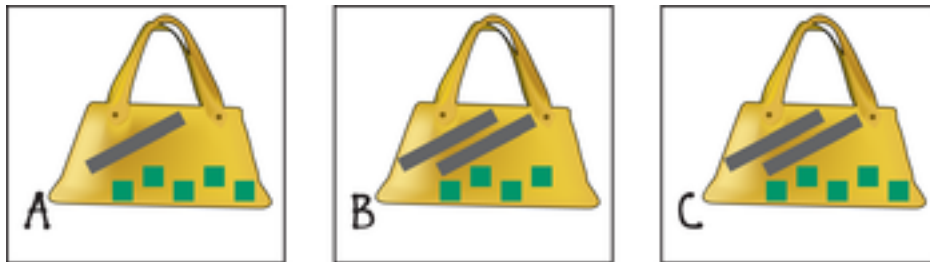
Answers:

1. Book A

2. Balloon B

3. Purse C

Explore More

1. Mrs. Sillysue's Mask has a yellow nose. Her mask has a green mouth. Which mask is Mrs. Sillysue's mask?
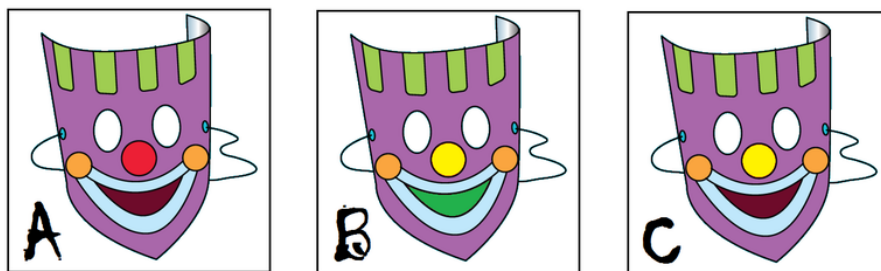
2. Jellybean's Shoes each have two eyes. Her shoes don't have teeth. Which shoes are Jellybean's shoes?
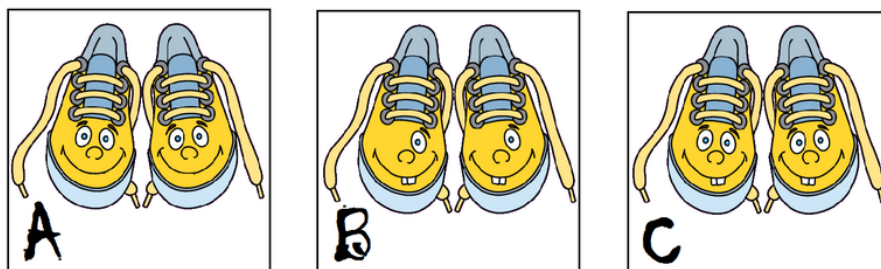
3. Mr. Candybob's Mask has a blue tooth. His mask has a green face. Which mask is Mr. Candybob's mask?
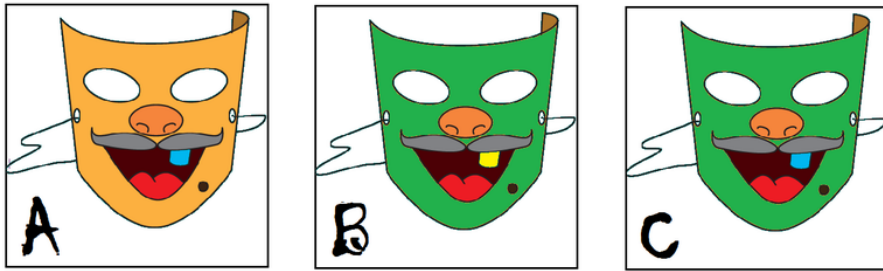
4. Connie's painting has two stars. Her painting has two moons. Which painting is Connie's painting?
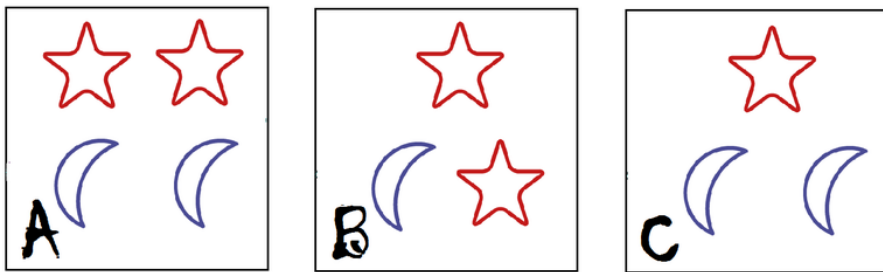
5. Edward the Elf's Shoes have blue pompoms. His shoes are purple. Which shoes are Edward the Elf's shoes?
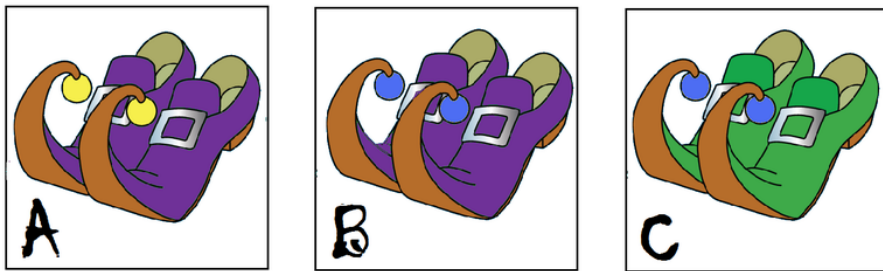
Summary

In these concepts we first learned to compare quantities and weights with picture graphs and teeter-totters. We learned to use patterns to figure out missing information with train cars. We also used proportional reasoning with packing bags. Last, we learned how to use clues and deductive reasoning to match pictures.

CHAPTER **2** # CK-12 Algebra Explorations Concepts, Kindergarten

Chapter Outline

Introduction

In these concepts, you will continue to develop the basic concepts of algebra and will practice your problem solving skills. There are six concepts, and each one focuses on a key algebraic thinking strategy. You will focus on describing, solving, and checking your thinking.

2.1 Number Towers

Students fill in missing numbers on number towers so that numbers are ordered from least to greatest.

What do you notice in the number tower below? What if you wanted to move the numbers in the cloud to the number tower so that the numbers were in order? In this concept, we will practice using deductive reasoning to fill in missing numbers.

Guidance

When looking at a number tower and cloud like the ones above, we can fill in the number tower by moving numbers from the cloud to the tower. We want the numbers in order so the least number is on the top and the greatest number is on the bottom. We can start at the top and find the smallest number and put it at the top of the number tower under the 3. Continue until we have filled in the whole number tower.

Example A

Use the numbers in the clouds. Fill in cubes with numbers. The least number is on the top and the greatest is on the bottom.

Solution:

Example B

Use the numbers in the clouds. Fill in cubes with numbers. The least number is on the top and the greatest is on the bottom.

Solution:

Example C

Use the numbers in the clouds. Fill in cubes with numbers. The least number is on the top and the greatest is on the bottom.

Solution:

Concept Problem Revisited

If we fill in the cubes with numbers so that the least number is on the top and the greatest is on the bottom, this is the solution:

Vocabulary

The *least number* is the smallest number. The *greatest number* is the biggest number. In the set of numbers {1, 6, 3, 9, 7}, 1 is the least number and 9 is the greatest number.

Guided Practice

Use the numbers in the clouds. Fill in cubes with numbers. The least number is on the top and the greatest is on the bottom.

1.

2.

2

6   8

3

9

3.

3

7   4

6

9

Answers:

2
5
6
7
8

1.

2
3
6
8
9

2.

3.
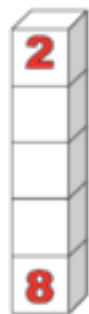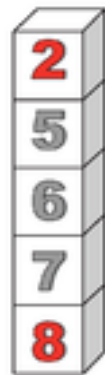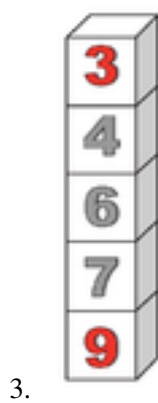
Explore More

Use the numbers in the clouds. Fill in cubes with numbers. The least number is on the top and the greatest is on the bottom.

1.

2.

3.

4.

5.
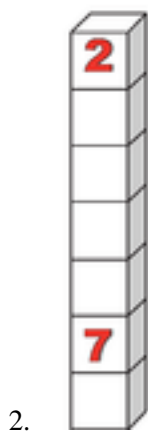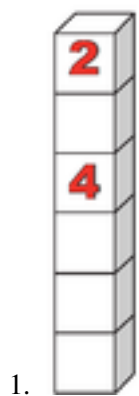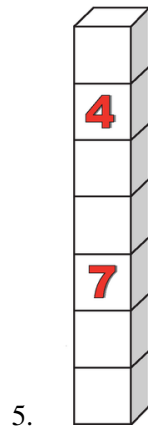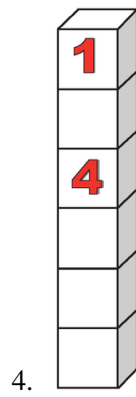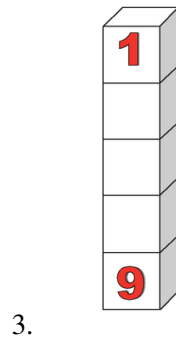
2.2 Straws and Glasses

Students practice counting by 2s and by 5s.

Can you draw 5

in each glass? Then can you count to figure out how many straws in all? Can you count by 5s? In this concept, we will practice drawing straws and then counting by 2s or 5s to figure out how many we have.

Guidance

Once you have drawn in straws like in the problem above, there are many ways to count how many we have. Since there are 5 straws in each glass, one way to count all of the straws is to count by 5s. If we put 2 straws in each glass, we could count all of the straws by counting by 2s.

Example A

Look at the picture below:

Draw 2

in the glass.

Your picture should look like this:

Count the straws. How many straws in all?

You should see that there are 2 straws in all.

Example B

Look at the picture below:

Draw 2

in each glass.

Your picture should look like this:
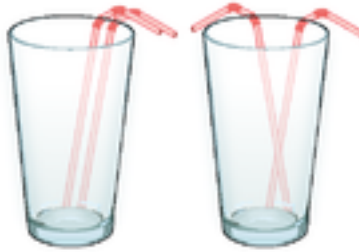
Count the straws. How many straws in all?

You should see that there are 4 straws in all. We can count by 2s ("2, 4") to get our answer.

Example C

Look at the picture below:

Draw 2

in each glass.

Your picture should look like this:

Count the straws. How many straws in all?

You should see that there are 8 straws in all. We can count by 2s ("2, 4, 6, 8") to get our answer.

Concept Problem Revisited

Once we draw 5 straws in each glass, our glasses look like this.

We can count by 5s to figure out how many straws there are in all. When we count like 5s it sounds like "5, 10, 15." There are 15 straws in all.

Vocabulary

Counting by 2s is when you count objects in groups of 2. When you count this way, it sounds like "2, 4, 6, 8, 10, ...". *Counting by 5s* is when you count objects in groups of 5. When you count this way, it sounds like "5, 10, 15, 20, 25, ...".

Guided Practice

Draw 5

in each glass. Count the straws. How many straws in all?

1.

_____ **straws in all**

2.

_____ **straws in all**

3.

_____ **straws in all**

Answers:

1. There are 5 straws in all.

2. There are 10 straws in all.

3. There are 20 straws in all.

Explore More

Draw 2

in each glass. Count the straws. How many straws in all?

1.

2.
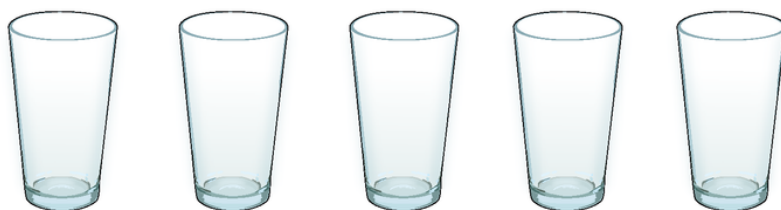
3.

Draw 5

in each glass. Count the straws. How many straws in all?

4.

5.

2.3 Teeter Totter

Students compare the weights of three animals shown in pairs on teeter totters.

Can you tell which animals weigh the same? Can you tell which animal weighs the least? In this concept, we will look at sets of teeter totters with animals on them and compare the weights of the animals.
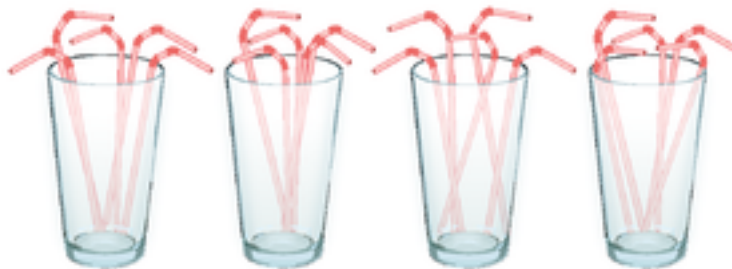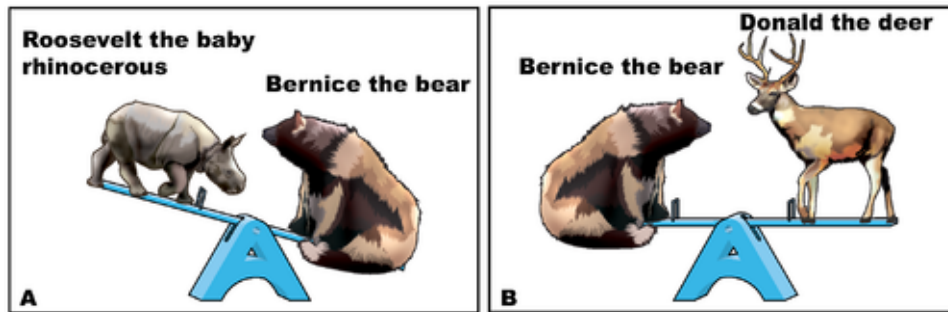

Guidance

When looking at a set of teeter totters like the one above, we learn different things from each picture. If the teeter totter is level, then we know that those two animals weigh the same. If the teeter totter is not level, then we know that the side lower to the ground has the animal that weighs more. We can combine this information to compare the weights of all three animals.

Example A

Look at the picture below. What do you see?
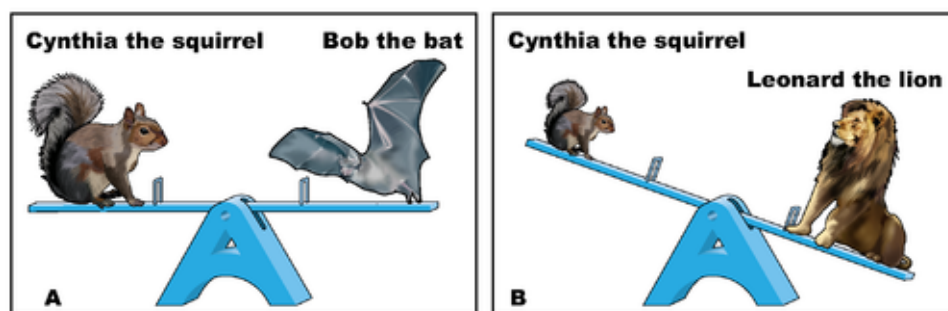
You should see a squirrel, a bat, and a lion. The squirrel is named Cynthia. The bat is named Bob. The lion is named Leonard.

Which two animals weigh the same?

You should see that the teeter totter is level in Picture A, so Cynthia and Bob weigh the same.

Which animal weighs more in Picture B? How can you tell from the picture?

You should notice that Leonard the lion weighs more because the lion side of the teeter totter is lower.

Who weighs the most?

Leonard the lion because the squirrel and the bat weigh the same. The lion weighs more than the squirrel, so he also weighs more than the bat. The lion weighs the most.

Example B

Look at the picture below. What do you see?

You should see a cat, a dog, and a raccoon. The cat is named Kit. The dog is named Donald. The raccoon is named Rafael.

Which picture shows that Kit the cat weighs less than Donald the dog? How can you tell from the picture?

We can see that Kit weighs less than Donald from picture B. This is because Donald the dog is on the bottom of the teeter totter and Kit the cat is on the higher side.

Which picture shows that Rafael the Raccoon weighs the same as Donald the Dog? How can you tell from the picture?

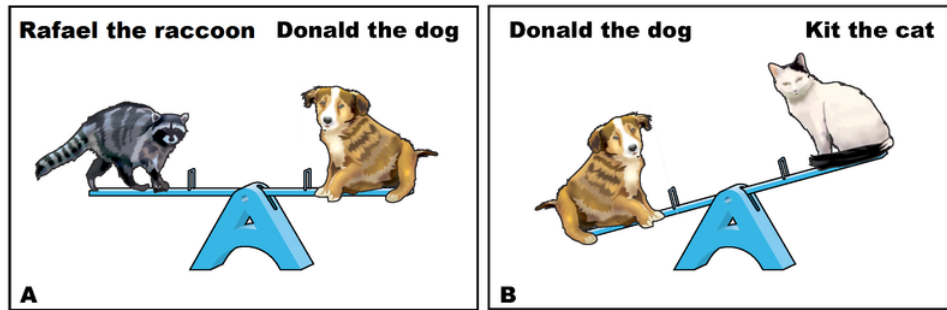We can see that Rafael weighs the same as Donald from picture A because the teeter totter is not tipped.

Who weighs the least? How did you decide?

Kit the cat because the cat weighs less than the dog. Since the raccoon and the dog weigh the same, the cat also weighs less than the raccoon.

Concept Problem Revisited
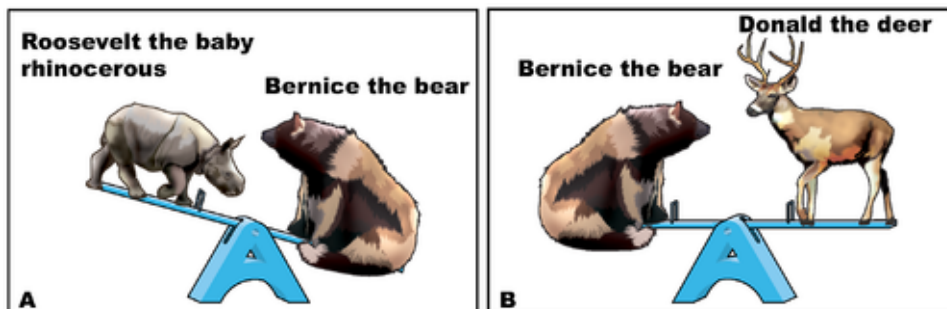
You should see Roosevelt the rhinoceros, Donald the deer and Bernice the bear on teeter totters. We can see the Roosevelt the rhinoceros weighs less than Bernice the bear from picture A. We can see that Roosevelt the rhinoceros weighs the same as Donald the deer from picture B. Roosevelt the rhinoceros weighs the least because he weighs less than Bernice the bear and Bernice the bear weighs the same as Donald the deer. So Roosevelt the rhinoceros weighs less than Donald the deer. Roosevelt weighs the least.

Vocabulary

To **weigh more** means to be **heavier**. To **weigh less** means to be **lighter**. We can compare the weights of objects by looking at them on a teeter totter.
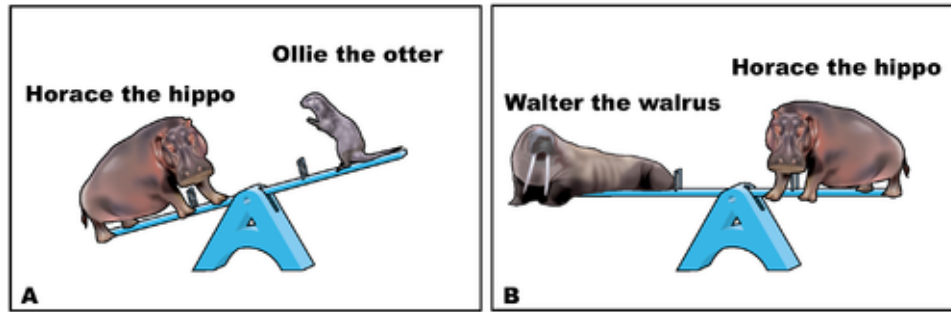
Guided Practice

1. Describe what you see in the picture.
2. Which two animals weigh the same? Which picture shows that they weigh the same?
3. Which animal weighs less than Horace the hippo? How can you tell from the picture?
4. Who weighs the least? How can you tell?

Answers:

1. We see Horace the hippo, Ollie the otter and Wally the walrus on teeter totters.
2. Wally the walrus and Horace the hippo which we see in picture B.
3. Ollie the otter because Ollie's side of the teeter totter is higher.
4. Ollie the otter since the walrus weighs the same as the hippo, and the hippo weighs more than the otter, then the walrus weighs more than the otter. So the otter weighs the least.

Explore More

1. Describe what you see in the picture.
2. Which two animals weigh the same? Which picture shows that they weigh the same?
3. Which animal weighs more than Orly the Otter? How can you tell from the picture?
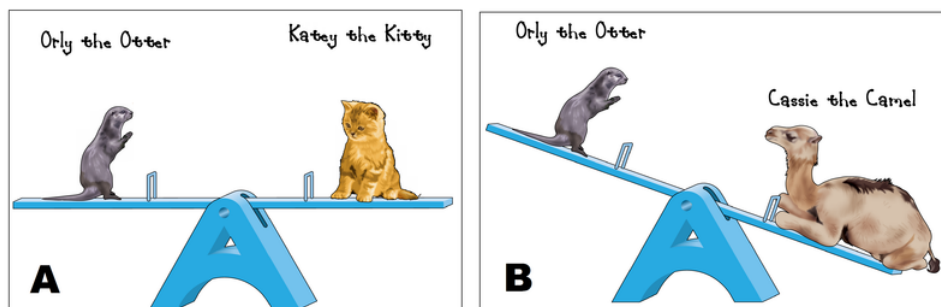4. Who weighs the most? How can you tell?
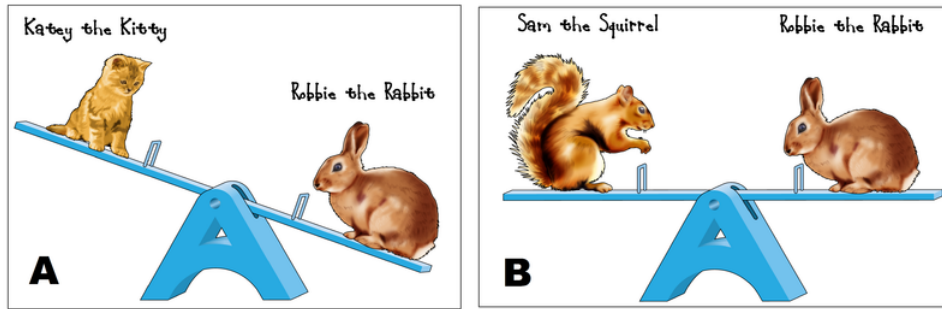
5. Describe what you see in the picture.
6. Which two animals weigh the same? Which picture shows that they weigh the same?
7. Which animal weighs less than Robbie the Rabbit? How can you tell from the picture?
8. Who weighs the least? How can you tell?

2.4 T-Shirt Mystery

Students use clues to determine the correct t-shirt from 4 choices.

Can you use clues to solve a mystery? If we know the following clues about Pansy Pelican's t-shirt, can we figure out which t-shirt is Pansy Pelican's?

Pansy Pelican's t-shirt has:

- **Three shapes with exactly 4 sides**
- **Two shapes with 6 corners**
- **One circle**

Here are the possible t-shirts:
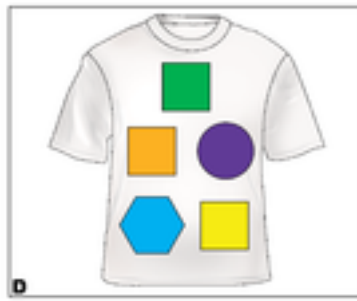
Guidance

In order to solve a mystery like the one above, we can use clues. First, read the first clue. See which t-shirts match the first clue. Then, read the second clue. See which t-shirts match the second clue. Continue as you go through all clues. The correct t-shirt will be the only one that works with all the clues.

Example A

Look at the t-shirts below. We want to figure out which t-shirt belongs to Lucy Goosie.
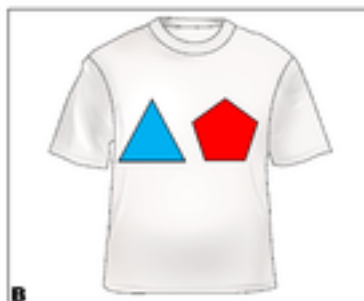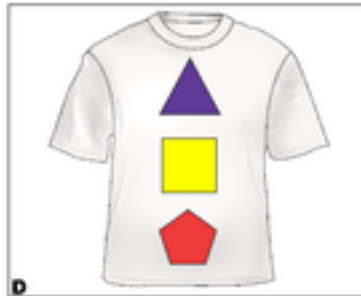
Our first clue is that Lucy Goosie's t-shirt has one square. Which t-shirts could be Lucy Goosie's?

You should notice that t-shirts A, C, and D all have one square. Any of those t-shirts could be Lucy Goosie's.

Our second clue is that Lucy Goosie's t-shirt has a shape with 3 sides on it. Which of the remaining t-shirts could be Lucy Goosie's?

You should notice that t-shirts A and D have shapes with 3 sides on them. Either of those t-shirts could be Lucy Goosie's.

Our last clue is that Lucy Goosie's t-shirt has one shape with exactly 5 corners on it. Which t-shirt is Lucy Goosie's?

You should notice that t-shirt D must be Lucy Goosie's. It is the only one between t-shirt A and D with a shape with exactly 5 corners on it.

Example B

Look at the t-shirts below. We want to figure out which t-shirt belongs to Edna Elephant.

Our first clue is that Edna Elephant's t-shirt has one shape with exactly 4 sides on it. Which t-shirts could be Edna Elephant's?

You should notice that t-shirts B, and C all have one shape with exactly 4 sides on it. Either of those t-shirts could be Edna Elephant's.

Our second clue is that Edna Elephant's t-shirt has two shapes with 3 corners on it. Which of the remaining t-shirts could be Edna Elephant's?

You should notice that both t-shirts B and C have shapes with 3 corners on them. Either of those t-shirts could be Edna Elephant's.

Our last clue is that Edna Elephant's t-shirt has exactly one circle. Which t-shirt is Edna Elephant's

You should notice that t-shirt B must be Edna Elephant's. It is the only one between t-shirt B and C with exactly one circle on it.

Concept Problem Revisited

Which T-shirt belongs to Pansy Pelican?

Clues: Pansy Pelican's T-shirt has:

- **Three shapes with exactly 4 sides**
- **Two shapes with 6 corners**
- **One circle**

From the first clue we know that Pansy Pelican's t-shirt must be either B, C, or D. From the second clue we know that her t-shirt must be either B or C. From the third clue we know that her t-shirt must be t-shirt C. Pansy Pelican's t-shirt is C.

Vocabulary

A *clue* is a piece of information that helps us to solve a puzzle or mystery. When we use clues, we are using *deductive reasoning*.

Guided Practice

Which T-shirt belongs to Sam Snake?
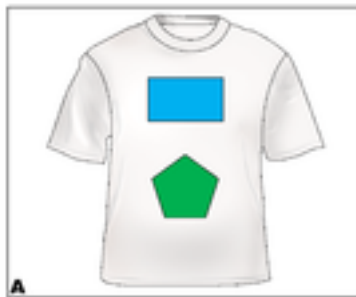
Clues: Sam Snake's T-shirt has:

- **Only one shape with exactly 5 corners**
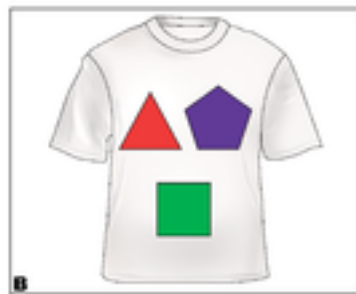- **Only one shape with 4 sides**
- **No triangles**

Answer:

From the first clue we know that Sam Snake's t-shirt must be either A or B. From the second clue we still know that his t-shirt must be either A or B. From the third clue we know that his t-shirt must be t-shirt A. Sam Snake's t-shirt is A.

Explore More

1. Which T-shirt belongs to Katey Kitty?

Clues: Katey Kitty's T-shirt has:

- Only one shape with exactly 4 sides
- Exactly two triangles
- Exactly one circle

2. Which T-shirt belongs to Orly Otter?
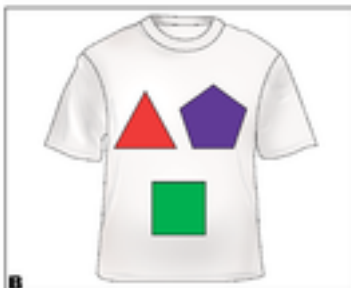
Clues: Orly Otter's T-shirt has:

- Exactly 5 shapes
- Only one shape with exactly 4 sides
- Exactly two triangles

3. Which T-shirt belongs to Andrea Aardvark?
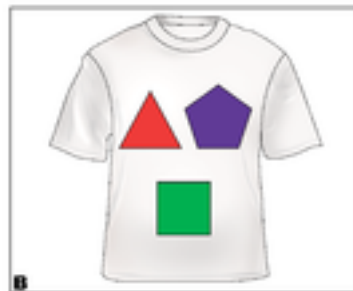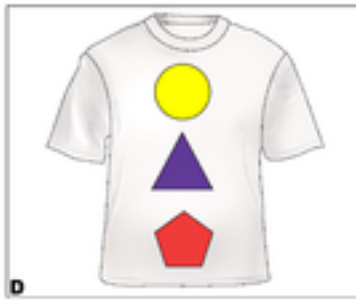
Clues: Andrea Aardvark's T-shirt has:

- Exactly one triangle
- Exactly three shapes
- Only one shape with exactly 4 sides

4. Which T-shirt belongs to Bernice Bear?
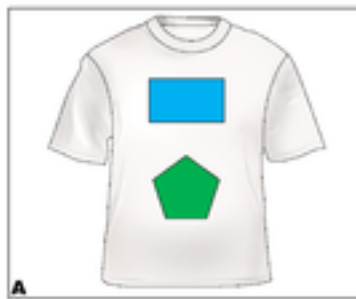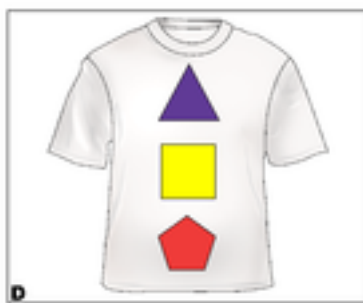
Clues: Bernice Bear's T-shirt has:

- Only one shape with exactly 4 sides
- Only one shape with exactly 5 sides
- Exactly one triangle

5. Which T-shirt belongs to Lester Lion?
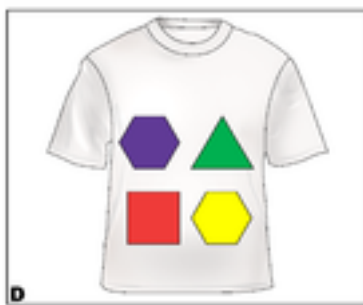
Clues: Lester Lion's T-shirt has:

- Two shapes with 6 sides
- No triangles
- Exactly three shapes with 4 sides

2.5 Sports Balls

Students interpret graphs and describe the relative number of sports balls that each person has.

The graph below shows the number of tennis balls that four different people have. Can you tell who has the most tennis balls? Can you tell who has the least tennis balls? Can you use the graph to figure out how many tennis balls each person has? In this concept, we will learn how to interpret picture graphs.
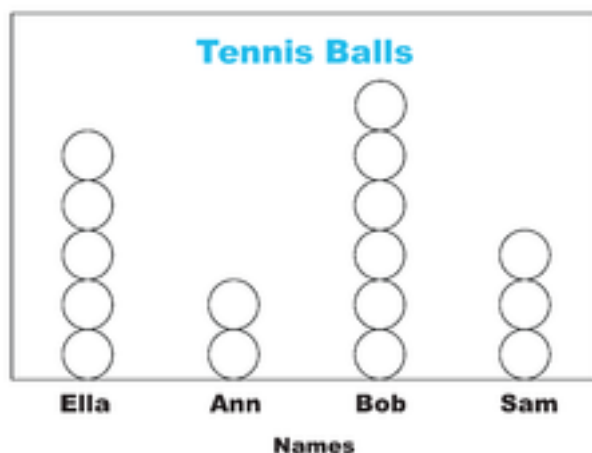
Guidance

In order to answer questions like the ones above about who has greatest number of sports balls and who has the least number of sports balls we can use the picture graph. The person with the greatest number of sports balls will have the highest stack. The person with the least number of sports balls will have the shortest stack. In order to figure out exactly how many sports balls each person has, we can count the number of sports balls in their stack.

Example A

Look at the picture graph below. This picture graph shows four students and the ping pong balls they have. The circles stand for ping pong balls. The students are Todd, Dina, Matt, Carol.
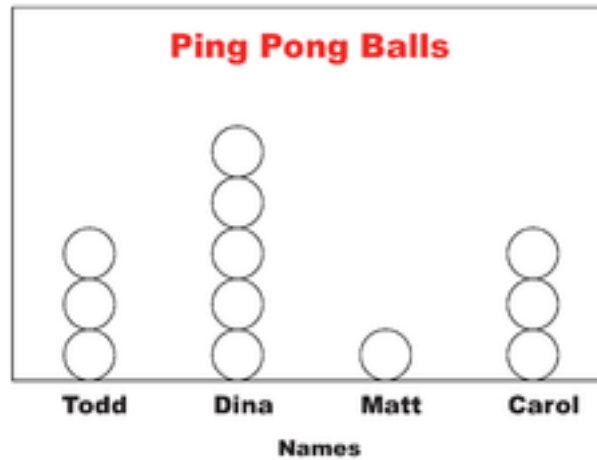
Ping Pong Balls

Matt has the least number of ping pong balls. How many ping pong balls does Matt have? Color Matt's ping pong balls blue.

You should see that Matt has 1 ping pong ball by counting the number of balls in his stack.

How can you tell from the graph that Matt has the least number of ping pong balls?

You can tell because his stack is the shortest.

Todd and Carol have the same number of ping pong balls. How many ping pong balls do they each have? Color their ping pong balls yellow.

You should see that Todd and Carol each have 3 ping pong balls.

How can you tell from the graph that Todd and Carol have the same number of ping pong balls?

You can tell because their stacks are the same height.

Dina has the greatest number of ping pong balls. How many ping pong balls does she have? Color Dina's ping pong balls red.

You should see that Dina has 5 ping pong balls.

How can you tell from the graph that Dina has the greatest number of ping pong balls?

You can tell because her stack is the highest. Here is a graph that has been colored.
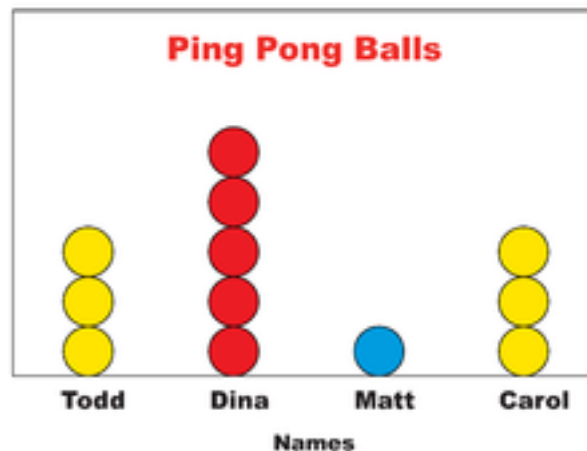
Ping Pong Balls

Example B

Look at the picture graph below. This picture graph shows four students and the ping pong balls they have. The circles stand for ping pong balls. The students are Jeff, Roz, Pedro, Cindy.
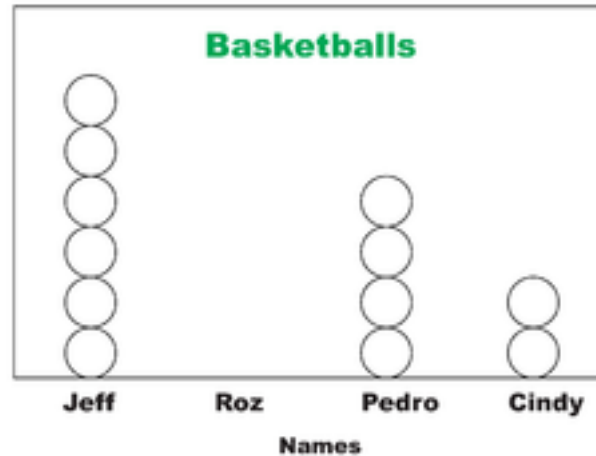
How many basketballs does Roz have?

Roz has zero basketballs. You can tell because there are no circles above her name.

Who has the greatest number of basketballs? How many basketballs does that person have? Color this person's basketballs red

Jeff has the greatest number of basketballs. He has 6 basketballs.

Pedro has 2 less basketballs than Jeff. How many basketballs does Pedro have? Color Pedro's basketballs green.

Pedro has 4 basketballs.

Cindy has 2 more basketballs than Roz. How many basketballs does Cindy have? Color Cindy's basketballs yellow.

Cindy has 2 basketballs. Here is a graph that has been colored.

Concept Problem Revisited
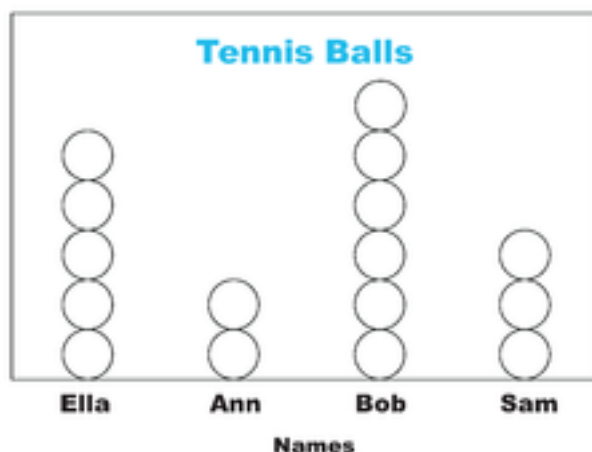
This picture graph shows four students and the tennis balls they have. The circles stand for tennis balls. The student's names are Ella, Ann, Bob, Sam.

From looking at the graph we can see that Bob has the greatest number of tennis balls because his stack is the highest. Bob has 6 tennis balls. We will color Bob's tennis balls green.
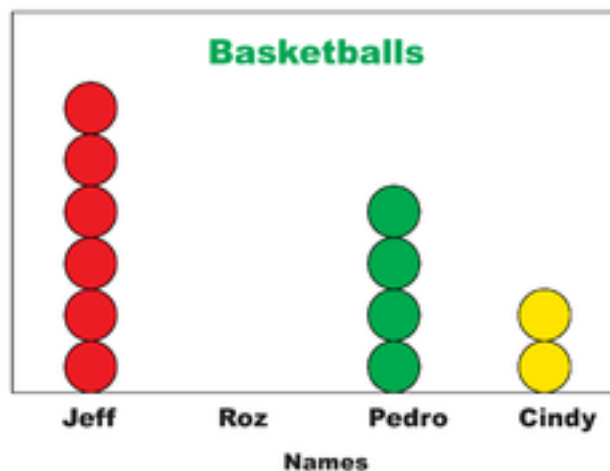
We can also tell from looking at the graph that Ann has the least number of tennis balls because her stack is the shortest. Ann has 2 tennis balls. We will color Ann's tennis balls yellow.

Sam has one more tennis ball than Ann, so Sam has 3 tennis balls. We will color Sam's tennis balls blue.

Ella has 5 tennis balls. We will color Ella's tennis balls red.

Here is what the graph looks like once it has been colored.
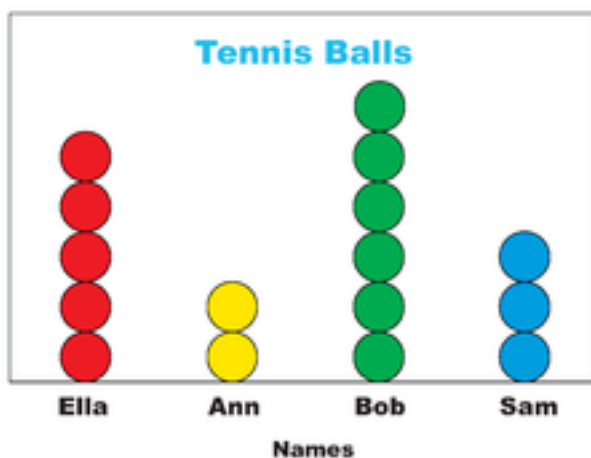
Vocabulary

To have *more* means to have a greater amount. The person with the *most* has the greatest amount. To have *less* means to have a smaller amount. To have the *least* means to have the smallest amount. In the picture below, Bob has the most tennis balls and Ann has the least tennis balls.

Tennis Balls

Ella Ann Bob Sam

Names
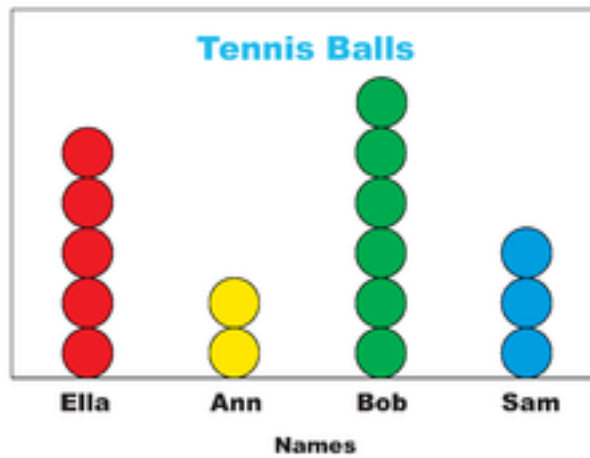
Guided Practice

This picture graph shows four students and the golf balls they have. The circles stand for golf balls. The students are Ang, Beth, Don, Eric.

Golf Balls

Amy Beth Don Eric

Names
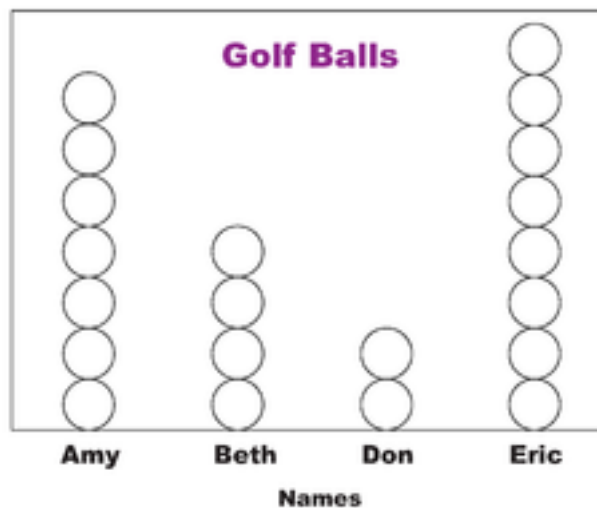
1. Who has the greatest number of golf balls? How many golf balls does this person have? Color this person's golf balls red.

2. Don has 6 less golf balls than Eric. How many golf balls does Don have? Color Don's golf balls yellow.

3. Beth has twice as many golf balls as Don. How many golf balls does Beth have? Color Beth's golf balls blue.

4. Ang has 3 more golf balls than Beth. How many golf balls does Ang have? Color Ang's golf balls green.

Answers:

1. Eric has the greatest number of golf balls. Eric has 8 golf balls.

2. Don has 2 golf balls.

3. Beth has 4 golf balls.

4. Ang has 7 golf balls.
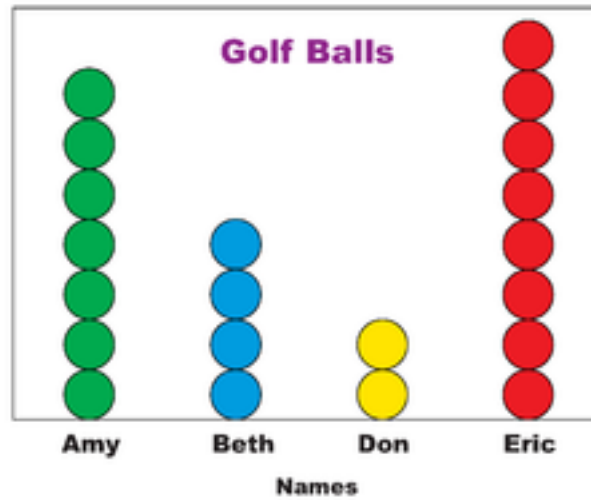
Explore More

This picture graph shows four students and the baseballs they have. The circles stand for baseballs. The students are Rochelle, Beth, Dan, and Rose.
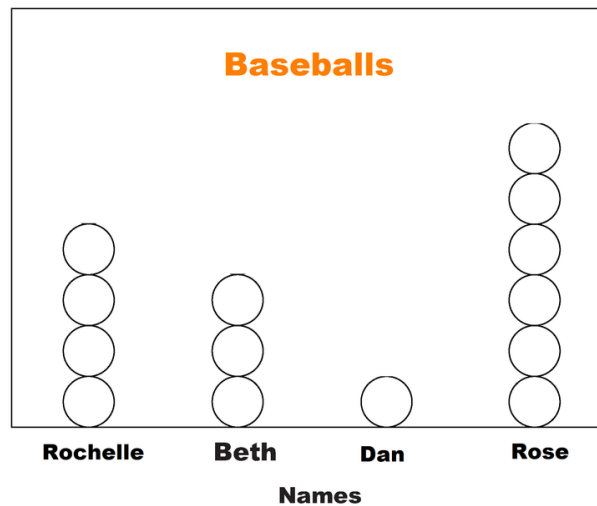
1. Who has the greatest number of baseballs? How many baseballs does this person have? Color this person's baseballs red.
2. Beth has 3 less baseballs than Rose. How many baseballs does Beth have? Color Beth's baseballs yellow.
3. Dan has 2 less baseballs than Beth. How many baseballs does Dan have? Color Dan's baseballs blue.
4. Rochelle has one more baseball than Beth. How many baseballs does Rochelle have? Color Rochelle's baseballs green.

This picture graph shows four students and the kickballs they have. The circles stand for kickballs. The students are Amanda, Tyler, Dan, and Nicole.

Kickballs

Amanda   Tyler   Dan   Nicole

Names
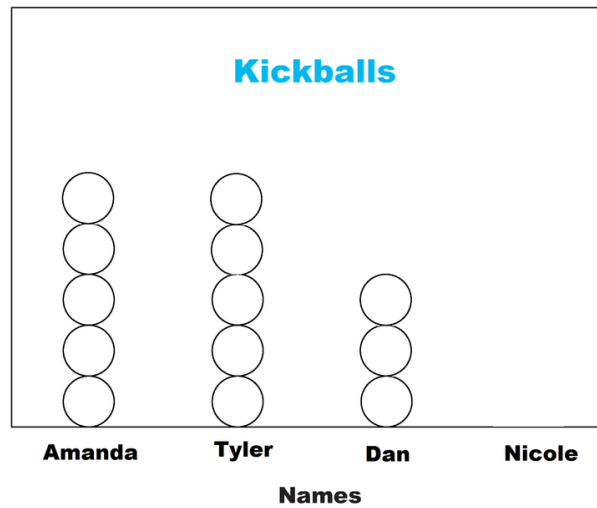
5. Who has the fewest number of kickballs? How many kickballs does this person have?
6. Amanda and Tyler have the same number of kickballs. How many do they each have? Color Amanda's kickballs blue and Tyler's kickballs yellow.
7. Dan has 2 less kickballs than Tyler. How many kickballs does Dan have? Color Dan's kickballs green.

2.6 Peculiar Patterns

Students identify and extend patterns.

Can you describe the pattern with the letters below? Can you write the next two rows of the pattern? In this concept, we will practice identifying patterns and extending them.

T M T

T N N T

T O O O T

Guidance

When identifying a letter pattern like the one above, the first thing we should do is describe anything we notice. For example, each line in the pattern has one more letter. Also, each line has a T at the beginning and at the end. Then, try to figure out what is changing from one line to the next.

Example A

Complete the next two rows.

Write the letters.

K

K L

K L M

K _____

K _____

Solution:

Each row in the pattern has one more letter. The letters are in the sequence of the alphabet. The fourth row should be KLMN. The fifth row should be KLMNO.

Example B

Complete the next two rows.

Write the letters.

C D

C D E

C D E F

_____

_____

Solution:

Each row in the pattern has one more letter. The rows always start with C, and then the letters are in the order of the alphabet. The fourth row should be CDEFG. The fifth row should be CDEFGH.

Example C

Complete the next two rows.

Write the letters.

F G H

F G H I

F G H I J

_____

_____

Solution:

Each row in the pattern has one more letter. The rows always start with F, and then the letters are in the order of the alphabet. The fourth row should be FGHIJK. The fifth row should be FGHIJKL.

Concept Problem Revisited

We saw the following pattern in the Concept section.

T M T

T N N T

T O O O T

In this pattern, each row has one more letter. Also, each row begins and ends with a T. In the middle of the T's, the number of letters goes up by 1 for each row. The letters in the middle of the T's are the next letter in the alphabet, starting with M.

The fourth row should be TPPPPT. The fifth row should be TQQQQQT.

Vocabulary

One type of *pattern* is when a characteristic (like letters) appear in a certain order. In the letter pattern below, each row has the next letter of the alphabet at the end.

C D

C D E

C D E F

Guided Practice

1. Complete the next two rows. Write the letters.

A B
B C C
C D D D
D _____

E _____

2. Complete the next two rows. Write the letters.

E E F
G G H
I I J

_____
_____

3. Complete the next two rows.Write the letters.

S O D
S O O D
S O O O D

_____
_____

Answers:

1. DEEEE; EFFFFF

2. KKL; MMN

3. SOOOOD; SOOOOOD

Explore More

1. Complete the next two rows. Write the letters.

X Y Z Z
X Y Z Z Z
X Y Z Z Z Z
X _____
X _____

2. Complete the next two rows. Write the letters.

M O O N
M M O O N
M M M O O N
M_____
M_____

3. Complete the next two rows. Write the letters.

R A R
R B B R
R C C C R
R _____
R _____

4. Complete the next two rows. Write the letters.

A B B
B C C
C D D

_____
_____

5. Complete the next two rows. Write the letters.

C A T
C A A T
C A A A T

_____
_____

Summary

In these concepts we used proportional reasoning when we filled glasses with straws and counted by 2s and 5s. We thought about equality and inequality when we compared weights in the teeter totter concept. We practiced deductive reasoning when figuring out the missing numbers in the number towers concept. We also practiced inductive reasoning when we looked for patterns and made predictions in the peculiar patterns concept. In all of the concepts, we practiced interpreting representations of mathematical relationships when we looked at towers of numbers, letters, patterns, and pictures of T-shirts with geometric shapes.