4.1 Indefinite Integrals Calculus

1. The antiderivative of 1 is x because the derivative of x is 1.

The antiderivative of $-3x^2$ is $-x^3$ because the derivative of $-x^3$ is $-3x^2$.

The antiderivative of $-6x$ is $-3x^2$ because the derivative of $-3x^2$ is $-6x$.

Thus, $F(x) = x - x^3 - 3x^2 + C$.

2. The antiderivative of x is $\frac{1}{2}x^2$ because the derivative of $\frac{1}{2}x^2$ is x.

The antiderivative of $-x^{\frac{2}{3}}$ is $-\frac{3}{5}x^{\frac{5}{3}}$ because the derivative of $-\frac{3}{5}x^{\frac{5}{3}}$ is $-x^{\frac{2}{3}}$.

Thus, $F(x) = \frac{x^2}{2} - \frac{3}{5}x^{\frac{5}{3}} + C$.

3. The antiderivative of $\sqrt[5]{2x+1} = (2x+1)^{\frac{1}{5}}$ is $\frac{5}{12}(2x+1)^{\frac{6}{5}}$ because the derivative of $\frac{5}{12}(2x+1)^{\frac{6}{5}}$ is $(2x+1)^{\frac{1}{5}} + C$.

4.

$$\begin{aligned}\int \left(2+\sqrt{5}\right) dx &= \int 2dx + \int \sqrt{5}dx \\ &= 2\int x^0 dx + \sqrt{5}\int x^0 dx \\ &= 2\left(\frac{x^{0+1}}{0+1}\right) + \sqrt{5}\left(\frac{x^{0+1}}{0+1}\right) \\ &= 2x + \sqrt{5x} + C\end{aligned}$$

5.

$$\begin{aligned}\int 2(x-3)^3 dx &= 2\int (x-3)^3 dx + \\ &= 2\frac{(x-3)^4}{4} + C \\ &= \frac{(x-3)^4}{2} + C\end{aligned}$$

6.

$$\begin{aligned}\int \left(x^2 \times \sqrt[3]{x}\right) dx &= \int x^{\frac{7}{3}} dx \\ &= \frac{x^{\frac{7}{3}+\frac{3}{3}}}{\frac{10}{3}} + C \\ &= \frac{3}{10}x^{\frac{10}{3}} + C\end{aligned}$$

7.

$$\begin{aligned}\int \left(x+\frac{1}{x^4\sqrt{x}}\right)dx &= \int \left(x+\frac{1}{x^{\frac{9}{2}}}\right)dx\\ &= \int xdx+\int x^{-\frac{9}{2}}dx\\ &= \frac{x^2}{2}+\frac{x^{-\frac{7}{2}}}{-\frac{7}{2}}+C\\ &= \frac{x^2}{2}-\frac{2}{7x^{\frac{7}{2}}}+C\end{aligned}$$

8.

$$\begin{aligned}f(x) &= \int 4x^3-3x^2+x-3dx\\ &= \int 4x^3dx-\int 3x^2dx+\int xdx-\int 3dx\\ &= 4\int x^3dx-3\int x^2dx+\int xdx-\int 3dx\\ &= 4\frac{x^4}{4}-3\frac{x^3}{3}+\frac{x^2}{2}-3\frac{x^1}{1}\\ &= x^4-x^3+\frac{x^2}{2}-3x+C\end{aligned}$$

9.

$$\begin{aligned}F(x) &= \int \left(2e^{2x}+x-2\right)dx\\ &= 2\frac{e^{2x}}{2}+\frac{x^2}{2}-2x+C\\ &= e^{2x}+\frac{x^2}{2}-2x+C\\ F(0) &= 5\\ F(0) &= e^{2(0)}+\frac{0^2}{2}-2(0)+C=1+C\\ 1+C &= 5\\ C &= 4\\ F(x) &= e^{2x}+\frac{x^2}{2}-2x+4\end{aligned}$$

10. $|x|=\begin{cases}x, x\geq 0\\ -x, x<0\end{cases}$

Then for $x \geq 0$,

$$\int |x|dx=\int xdx=\frac{x^2}{2}+C.$$

Also, for $x<0$,

$$\int |x|dx=\int -xdx=-\frac{x^2}{2}+C.$$