4.5 Evaluating Definite Integrals

1.

$$\int_{4}^{9} \left(\frac{3}{\sqrt{x}} \right) dx = \int_{4}^{9} 3x^{-\frac{1}{2}} dx$$

$$= 3 \int_{4}^{9} x^{-\frac{1}{2}} dx$$

$$= 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \Bigg]_{4}^{9}$$

$$= 6 \left(\sqrt{9} - \sqrt{4} \right)$$

$$= 6 (3 - 2)$$

$$= 6$$

2.

$$\int_{0}^{1} \left(t - t^{2} \right) dt = \int_{0}^{1} t \, dt - \int_{0}^{1} t^{2} \, dt$$

$$= \frac{t^{2}}{2} \Bigg]_{0}^{1} - \frac{t^{3}}{3} \Bigg]_{0}^{1}$$

$$= \left(\frac{1^{2}}{2} - 0 \right) - \left(\frac{1^{3}}{3} - 0 \right)$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

3.

$$\int_{2}^{5}\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{2}}\right)dx = \int_{2}^{5} x^{\frac{-1}{2}}\,dx + \int_{2}^{5}\frac{1}{\sqrt{2}}dx$$

$$= \left.\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{2}^{5} + \left.\frac{1}{\sqrt{2}}x\right]_{2}^{5}$$

$$= \left.2\sqrt{x}\right]_{2}^{5} + \left.\frac{1}{\sqrt{2}}x\right]_{2}^{5}$$

$$= \left(2\sqrt{5}-2\sqrt{2}\right) + \left(\frac{5}{\sqrt{2}}-\frac{2}{\sqrt{2}}\right)$$

$$= 2\sqrt{5}-2\sqrt{2}+\frac{3\sqrt{2}}{2}$$

4.

$$\int_{0}^{1}\left(x^2-1\right)\left(x^2+1\right)dx = 4\int_{0}^{1}\left(x^4-1\right)dx$$

$$= 4\left[\left.\frac{x^5}{5}\right]_{0}^{1} - x\big]_{0}^{1}\right]$$

$$= 4\left[\frac{1}{5}-1\right]$$

$$= \frac{4}{5}-4$$

$$= \frac{16}{5}$$

5.

$$\int_{2}^{8}\left(\frac{4}{x}+x^2+x\right)dx = \int_{2}^{8}\frac{4}{x}dx + \int_{2}^{8}x^2 dx + \int_{2}^{8}x\,dx$$

$$= 4\ln x\big]_{2}^{8} + \left.\frac{x^3}{3}\right]_{2}^{8} + \left.\frac{x^2}{2}\right]_{2}^{8}$$

$$= 4\ln 8 - 4\,4\ln 2 + \frac{512}{3} - \frac{8}{3} + 32 - 2$$

$$= 203.55$$

6. $\int_{2}^{4} e^{3x}dx = \left.\frac{e^{3x}}{3}\right]_{2}^{4} = \frac{e^{12}}{3} - \frac{e^{6}}{3}$

7.

$$\int_{1}^{4} \frac{2}{x+3}\,dx = 2\ln(x+3)\Big]_{1}^{4}$$

$$= 2\ln(7) - 2\ln(4)$$

8.

$$F(9) - F(1) = \int_{1}^{9} \sqrt{x}\,dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big]_{1}^{9} = \frac{2}{3}\left(\sqrt{x}\right)^{3}\Big]_{1}^{9} = \frac{2}{3}\left[\left(\sqrt{9}\right)^{3} - \left(\sqrt{1}\right)^{3}\right] = \frac{2}{3}(27-1) = \frac{52}{3}$$

$$\frac{F(9) - F(1)}{9 - 1} = \frac{\frac{52}{3}}{8} = \frac{52}{24} = \frac{13}{6}$$

9. By the Mean Value Theorem for derivatives there exits a c in $[a,b]$ such that

$$\int_{1}^{4} f(x)\,dx = f(c)(b-a)$$

$$9 = f(c)(4-1)$$
$$9 = f(c)(3)$$
$$3 = f(c)$$

10. He is partially correct. The definite integral $\int_{0}^{2\pi} \sin x\,dx$ computes the net area under the curve. However, the area between the curve and the $x-$axis is given by $\int_{0}^{2\pi} \sin x\,dx - \cos x]_{0}^{\pi} = 2$.