14.01SC Principles of Microeconomics, Fall 2011
Transcript – Problem 3-5 Solution Video

PROFESSOR: Hi, and welcome back to the 14.01 problem solving videos. Today, we're going to do Fall 2010, Problem Set 3, Problem Number 5. And we're going to go ahead and we're going to work through parts A, B, C, D, and E, and then we're going to finish up parts F and G.

Problem 5 says that Xiao spends all her income on statistical softwares and clothes. Her preferences can be represented by the utility function where her utility equals 4 times the natural log of S plus 6 times the natural log of C, where S is software and C is clothes. Part A asks us to compute the marginal rate of substitution of software for clothes, asks us if the MRS is increasing or decreasing in S, and also asks us how we interpret the MRS.

So before we start with this, we should really think about, conceptually, what the marginal rate of substitution of software for clothes looks like on our graph. So on this graph, I have clothes on the y-axis-- the quantity of clothes-- and the quantity of software on the x-axis here. Looking at this graph, this line that I've drawn is one utility level. So at this place, she might have a utility equal to 1. So she's indifferent on this indifference curve between being at point here or at a point here.

And what the marginal rate of substitution is really asking us, it's asking us how much clothing is she willing to give up to get one more unit of software? So she's going to have to give up a certain amount of clothes to get one more unit of software. And the marginal rate of substitution tells us exactly how much clothing she's willing to give up.

To calculate this algebraically, all we're going to do is we're going to take the marginal utility of software and divide it by the marginal utility of clothes. So we're going to take the derivative with respect to software and the derivative with respect to clothing and divide. When we do this, we find that the MRS is going to be equal to 4 over S, which is our marginal utility of software, all over 6 divided by C, which is our marginal utility of clothes. Solving through, we find that our MRS is 4C over 6S.

Now, we have to think about, conceptually, what happens when software increases? When we have S increase, since it's in the denominator, we're also going to have the MRS decrease. So what this means is as software is increasing, or as she has more software, she's going to be willing to give up fewer clothing, or less clothing, to get another unit of software. So looking at our graph, when she's at this point, she's more willing to give up clothing to get more software. But when she has more software down here, she's less willing to give up the clothing.

Let's go ahead and move on to Part B. Part B, find Xiao's demand functions for software and clothes-- so we're going to call those QS and QC-- in terms of the price of software PS, the price of clothes PC, and Xiao's income. Now, before we move on with this, what we want to do is we want to solve for one of the variables C or S in terms of the prices and the other variable. So to do this, we're going to set the MRS equal to the price of the software over the price of the clothes.

From here, we can solve through for C, and we find that C is going to be equal to 3/2 times PS over PC times S. Now, since we have two variables-- we have a variable for clothes and a variable for software-- we're going to have to introduce another constraint into this problem. And the constraint that we're going to introduce is going to be the income function. We know that Xiao has some sort of income that's going to be fixed, and she's going to spend all of this on either clothes or software.

Now, the amount of money she spends on software is going to be equal to the price of software times how much software she's going to buy. The rest of her income is going to be spent on clothes, so the price of clothes times the quantity of clothing. Now, to solve for the demand function for software, all we're going to do is we're going to plug in for C in the income function here, and then we're going to solve through for S.

I know it's a little bit messy, but this says PS times 3/2PS over PC, what we solved for here, times S. Now, when we solve through for S from this equation, we're going to find that the demand function for software is going to be equal to 2/2 times income over the price of software. Now, we can go through the same process solving for the demand function for clothing. And all we'd have to do now is we can take this S right here that we just solved for, we can plug this back into our income function, and then we can solve for C. When we solve for C, we're going to find that the demand function for clothing is going to equal 3/5 times I over PC.

Part C asks us to draw the Engel curve for software. Now, all an Engel curve is, it's a relationship between the income and the quantity that's demanded for a product. And it shows us that as income increases, it shows how the quantity demanded is going to change with changing income. So to start off our Engel curve, we're going to draw an axes, we're going to put software, or the quantity that's demanded, on the x-axis, and we're going to put the income on the y-axis. Now, the nice thing about the software demand function that we solved for is that it's linear with respect to income.

Now, before we can graph this equation, however, we have to get it in terms of income. So when we solve for this, we're going to find that income equals 5/2 PS times S. So all our Engel curve is going to look like is it's going to be a straight line. And the slope of that straight line is going to be 5/2 PS. And the way to interpret this conceptually is to say that with each one unit increase in income, the amount that's demanded is going to increase by 2/5 divided by PS.

Let's go ahead and move on to Part D. Part D says, suppose that the price of software is PS equals 2, and the price of clothing is going to equal PC equals 3. And Xiao's income is going to equal 10. What bundle of software and clothes maximize Xiao's utility? Now, we've already found the conditional demand curves for both software and clothes.

So we can start off this problem by writing down those conditional demand curves. The conditional demand curve for software was given by 2/5 I divided by PS. And the conditional demand for clothing was given by 3/5 I divided by PC. All we have to do now is we have to plug in these variables to solve for the software and the clothing that's going to be demanded. When we plug those in, we're going to find that she's going to demand two units of both software and clothes.

So this is in the scenario for Part D. Part E gives us another scenario that we can solve for. And all that's going to happen now is that the price of software is going to change. And we're going to look at how that affects the bundle that maximizes her utility. For Part E, it says, suppose that the price of software increases from PS equal to 2, and now it's going to be PS is going to equal 4. What bundle of software and clothes does Xiao demand now?

Again, we're just going to solve through. With our new PS equals 4, we're going to solve for the software and clothing that Xiao demands. We're going to find that S is going to equal 1 now, and that the amount of clothing is going to equal 2. So let's take a pause right here. And we're going to come back in just a minute.

And we're going to look at the more interesting case, which is given the fact that she's consuming less-- she has one less unit of software to consume-- how do we get her back to the utility that she had before? What amount of money or income do we have to give her so that she can be as happy as she was in this initial scenario with two units of both software and clothes?

Welcome back. So we're going to continue onto Part F. Part F says, given the price increase, how much income does Xiao need to remain as happy-- have the same utility-- as she was before the price change? What bundle of softwares and clothes would Xiao consume if she had the additional income given the new prices? So we want to find out, how can we give her as much income so she can be as happy as she was to start off with?

To start this problem, the first thing that we're going to have to find out is we're going to have to find out exactly how happy Xiao was to begin with. So we need to know her initial utility. So let's start off with that calculation. To calculate her initial utility, we're just going to start off by saying that her utility is equal to 4 natural log of S plus 6 natural log of C. And we can plug in 2 and 2 for S and C. In this case, when we solve through, we find that her initial utility is 6.931.

So we're going to set her utility equal to 6.931. And what we want to find out is we want to find out what income we have to give her so that she can get up to this utility, given the new prices. So we're going to take this utility function, and we're going to plug in the conditional demand curves for S and C so that income is now a function, or one of the inputs for her utility.

When we do that, we're going to get this function. And remember, we said that we want, given this input and the new prices-- so we're going to set this PS is going to be equal to the new price in the problem. And we're going to also set the PC equal to the price that was in Part E as well.

And flipping back to the problem, we know that the price of clothing is going to be equal to 3, and the price of software for the second part was equal to 4. So we're going to plug in for PS and PC, we're going to set utility equal to 6.931, and we're going to solve through for I. When we do this, and when we solve through for I, what we're going to find is we're going to have 6.931 is going to be equal to 10 natural log of I minus 4 natural log of 10 minus 6 natural log of 5.

Solving through, doing the inverse natural log function for I after isolating this variable, we're going to find that the new income that she needs to be supplied is 13.19. So the income that she needs to be just as happy with these prices has increased by 3.19. Now, we can go back to our conditional demand curves that we had here. We can plug in PS equals 4, PC equals 3, and we can plug in for income 13.19. And we can solve S double prime, which is the new software that she's going to demand, which will be 1.32, and C double prime, the new amount of clothes that she's going to demand, which is 2.64.

Now, the final part of the problem, which we're going to move on to now, which is Part G, is actually the most important part of the problem, because what we're going to do is we're going to tie together the three scenarios that we did. We did this scenario where we were giving her income so that she would be just as happy. We had our initial scenario before the price increase. And we had the scenario after the price increase.

And we're going to look at this conceptually on a graph, and we're going to see, how do we relate these three bundles of consumption? Part G says, going back to the situation in Part E, where PS equals 4 and I equals 10, we need to decompose the total change of softwares and clothes demanded into substitution and income effects. In a clearly-labelled diagram, with softwares on the horizontal axis, show the income and substitution effects of the increase in the price of software.

Now, we're going to go back to this graph that we started off with. And what we're going to do here, is we're going to illustrate the three bundles that she selected. I'll illustrate the first bundle where she consumes 2 units of each. And we already have our utility curve, or indifference curve, drawn up here. Now we need to draw the budget constraint that shows how much she can spend on each product.

If she were to spend all her money on clothes, she would be up here at this corner solution. If she were to spend all her money on software, she would be down here. When we connect a line through here, this is her budget constraint that shows all the possible bundles of goods where she could potentially spend her money. And this first bundle is the point 2,2. This is where she starts off to begin with.

Now, when the price of software increases, she's not going to be able to buy as much software with her money. But she can still buy the same amount of clothing. So her new budget constraint in this scenario is going to look like this. So in this scenario, which is in our problem's Part E, her utility has moved in towards the origin. And she isn't going to be as happy. And we can represent this on a utility curve as well.

And we can see from this point on the utility curve the way that I've drawn it, that at this point, she's still consuming the same amount of clothing. But the amount of software she's consumed has been cut in half. This is the total effect of the price change. It's the difference between where she started and where she's ending up without giving her any money to change where she actually is. So the total effect is just that she's losing one unit of software.

Now, we can break down the total effect in the substitution effects and income effects. It's important that we really understand conceptually how to define substitution and income effects. And what we're going to think about is we're going to think about, on this graph, we're going to represent the substitution effect by the movement-- if we were to just have the price change, and we were to give her income so she could stay up at this utility level, the substitution effect is how her bundle changes with that movement.

The income effect is going to be-- since she's poorer because the prices are higher, it's going to be the shift downward. So I'm going to draw in the scenario that we calculated in Part F right here. By drawing in this scenario with the higher income level and the price change, we can represent this bundle as the substitution effect.

So what this looks like is she's going to have the same budget constraint, only it's going to be shifted back up. This is going to be the bundle in Part F. And we can label the bundle 1.32, 2.64. And this is where it's going to get a little bit tricky. The substitution effect is just the movement from 2.2 to the same utility curve but at a different point with a different bundle. So it's when we've given her income to keep her at the same utility level, but we've had the price change.

This is going to be the substitution effect. I'm going to label it 1. Now, the income effect is the next movement. It's the movement that says, well, we don't really give her more income. She's actually poorer. It's the movement down from 1.32, 2.64, down to 1, 2. And then the total effect is just this movement from 2.2 to 1.2. So I can label the substitution effect 1, the income effect 2, and the total effect 3.

So to calculate the substitution effect, all it's going to be is it's going to be the difference between 2.2 and 1.32 and 2.64. So in this case, our substitution effect is going to be equal to 0.68, negative 0.64. And you can see that we actually had an increase in the consumption of clothes for the substitution effect.

And then the income effect, using this equation and what we calculated the substitution effect and the total effect to be, we find that the income effect is equal to 0.32, 0.64.

So what this problem basically had us do is it made us look at the effect of a price change on the consumption decisions of a consumer. So when a price increases, two things are basically happening. The first thing that's happening is the price of that product is more, so the person, in most cases, shifts their buying away from that product and towards the less expensive product. That's what this substitution effect shows us.

The other effect that the person feels is since the price is higher, they can't buy as much stuff with the money that they have. So they feel poorer, even though they have the same amount of money, because the prices are higher. That's what this income effect represents. And the total effect is just the summation of the fact that the price is higher for one good and that they feel poorer. And so we looked at the total effect broken down into substitution and income effect.