14.01 Fall 2010
Problem Set 7 Solutions

1. (20 points) For each of the following statements, please indicate whether they are TRUE, FALSE, or UNCERTAIN. No credit will be given without an explanation as to why your claim is true.

    (a) (5 points) If a monopsonist faces a perfectly elastic supply curve, there will be no deadweight loss relative to the competitive outcome.

    True. If a monopsonist faces a perfectly elastic supply, then marginal expenditure is identical to average expenditure (the supply curve). Thus, the quantity will be set at the point where both marginal and average expenditure equal marginal value (the competitive outcome).

    (b) (5 points) It is economically more efficient to have a monopolist that discriminates perfectly than a monopolist that sets a single price.

    True. When a monopolist discriminates perfectly, the quantity supplied is equal to the quantity supplied in a competitive market, and the total producer surplus is equal to the sum of the consumer and producer surplus in a competitive market, therefore there is no DWL. Meanwhile, in class it was shown that when a monopolist sets a single price, there is DWL relative to the competitive outcome.

    (c) (5 points) In a Cournot duopoly market, the two firms agree to produce half of the monopoly output level for that market and split the resulting profit. Since the monopoly profit is the highest profit that can be obtained, the two firms will always stick to that agreement even if it's not legally (or in any other way) binding.

    False. If one firm is producing half of the monopoly output, it is individually rational for the other firm to produce more than half of the monopoly output and earn a profit that is higher than half of the monopoly profit it would be getting under the agreement. Hence, such an agreement cannot be sustained without some sort of enforcement.

    (d) (5 points) A firm is a monopolist in the market for good X. The government has perfect information about the marginal and average cost curves of this firm and also has perfect information about the demand curve for good X. Claim: The economy will reach an efficient outcome if the government sets a price ceiling that makes the price equal to the marginal cost, evaluated at the quantity where the marginal cost intersects the demand curve.

    Uncertain. Consider the case of a natural monopoly where the marginal cost is smaller than the average cost for all quantities. Assume the government sets a price ceiling that makes the price equal to the marginal cost, evaluated at the quantity where the marginal cost intersects the demand curve. The price will be lower than the average cost and the firm would eventually go out of business (unless the government covers the losses). Having the firm go out of business cannot be efficient, since all surplus is lost.

2. Problem solution removed due to copyright restrictions. This content is presented in audio form in the Solution Video for Problem Set 7, Problem 2.

3. (20 points) A monopoly faces market demand $Q = 30 - P$ and has a cost function $C(Q) = \frac{1}{2}Q^2$.

    (a) (5 points) Find the profit maximizing price and quantity and the resulting profit to the monopoly.

    The monopoly produces at the point where $MR = MC$. In this question $MR = \frac{d(PQ)}{dQ} = 30 - 2Q$ and $MC = \frac{dC}{dQ} = Q$. Equating MR and MC gives us $Q = 10$. From the demand equation we can find $P = 30 - Q = 20$. Profit $= PQ - C(Q) = 20 \cdot 10 - 50 = 150$.

    (b) (5 points) What is the socially optimal price? Calculate the deadweight loss (DWL) due to the monopolist behavior of this firm. Calculate consumer surplus (CS) and producer surplus (PS). Show CS, PS, and DWL on the diagram.

    We can find the socially optimal price and quantity by equating demand and MC: $30 - Q = Q$. That means $Q = 15$ and $P = 30 - Q = 15$. Instead, monopoly sells $Q = 10$ at $P = 20$, which generates $DWL = (20 - 10)(15 - 10)/2 = 25$. $CS = (30 - 20) \cdot 10/2 = 50$ and $PS = (20 \cdot 10) - 10^2/2 = 150$. Note that the producer surplus is equal to the profit, in the absence of the fixed cost.
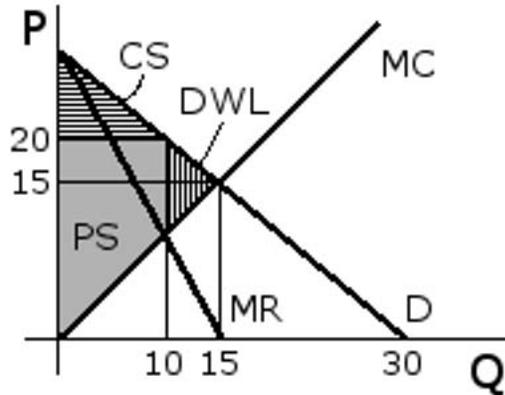
(c) (5 points) Assume that the government puts a price ceiling on the monopolist at $P = 18$. How much output will the monopolist produce? What will be the profit of the monopolist? Calculate CS, PS, and DWL. Why is the deadweight loss different now?

Now monopoly can't charge its optimal price of \$20. Instead, it chooses the maximum price it is allowed to charge: $P = 18$. Monopoly sells $Q = 30 - 18 = 12$ at this price. The resulting profit is $18 \cdot 12 - 12 \cdot 12/2 = 144$. $CS = 12 \cdot 12/2 = 72$, $PS = $ profit $= (18 + 6) \cdot 12/2 = 144$, and $DWL = 6 \cdot 3/2 = 9$. The DWL is smaller now because price is lower than under monopoly and output is higher.

(d) (5 points) Assume that the government put a price ceiling on the monopolist in order to maximize the total (i.e. consumer plus producer) surplus. What price ceiling should it choose? How much output will the monopolist produce at this price ceiling? What will the profit of the monopolist be? What is the DWL?

We know from part (b) that the socially optimal price is \$15. If monopoly is to sell at this price, then total surplus will be maximized and there will be no deadweight loss. Thus the government has to set maximum price equal to \$15. The monopoly will sell $Q = 30 - 15 = 15$ at the price $P = 15$. Profit $= 15 \cdot 15/2 = 112.5$, and $DWL = 0$.

<div align="center">Problem 3 solution courtesy of William Wheaton. Used with permission.</div>

4. (15 points) Suppose an industry has a duopoly structure. Duopolist 1 has a cost function given by:

$$c_1(y_1) = (y_1)^2 \text{ for } y_1 \geq 0$$

Duopolist 2 has a cost function given by:

$$c_2(y_2) = 12y_2 \text{ for } y_2 \geq 0$$

Denoting total output produced in the industry by $y = (y_1 + y_2)$, the inverse demand function for the good produced in the industry is given by:

$$p = 100 - y$$

(a) (5 points) Find the reaction function of each duopolist.

For Firm 1:

$$\max \pi_1 = (100 - y)y_1 - y_1^2 = (100 - y_1 - y_2)y_1 - y_1^2$$
$$\frac{\partial \pi_1}{\partial y_1} = -y_1 + 100 - y_1 - y_2 - 2y_1 = 0 \longrightarrow 4y_1 + y_2 = 100$$

For Firm 2:

$$\max \pi_2 = (100 - y)y_2 - 12y_2 = (100 - y_1 - y_2)y_2 - 12y_2$$
$$\frac{\partial \pi_2}{\partial y_2} = -y_2 + 100 - y_1 - y_2 - 12 = 0 \longrightarrow y_1 + 2y_2 = 88$$

<div align="center">2</div>

Reaction functions are as follows:

$$\text{Firm 1: } 4y_1 + y_2 = 100$$
$$\text{Firm 2: } y_1 + 2y_2 = 88$$

(b) (5 points) Using (a), obtain the output levels that will be produced in a Cournot-Nash equilibrium, and the price level in such an equilibrium.

Solving the system of reaction functions we just found, we find:

$$y_1^* = 16, \quad y_2^* = 36 \quad \longrightarrow \quad y^* = 52$$

$$p^* = 100 - y^* = 48$$

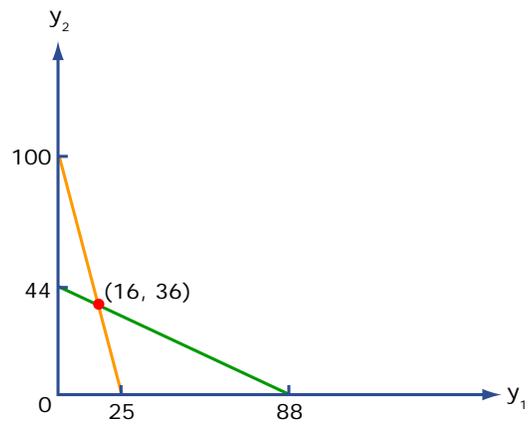(c) (5 points) Illustrate your solution in (b) above in a suitable diagram.

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14.01SC Principles of Microeconomics
Fall 2011