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18.01 Single Variable Calculus
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Exam 1 Review

General Differentiation Formulas

$$\begin{aligned}(u + v)' &= u' + v' \\ (cu)' &= cu' \\ (uv)' &= u'v + uv' \quad (\text{product rule}) \\ \left(\frac{u}{v}\right)' &= \frac{u'v - uv'}{v^2} \quad (\text{quotient rule}) \\ \frac{d}{dx}f(u(x)) &= f'(u(x)) \cdot u'(x) \quad (\text{chain rule})\end{aligned}$$

You can remember the quotient rule by rewriting

$$\left(\frac{u}{v}\right)' = (uv^{-1})'$$

and applying the product rule and chain rule.

Implicit differentiation

Let's say you want to find y' from an equation like

$$y^3 + 3xy^2 = 8$$

Instead of solving for y and then taking its derivative, just take $\frac{d}{dx}$ of the whole thing. In this example,

$$\begin{aligned}3y^2y' + 6xyy' + 3y^2 &= 0 \\ (3y^2 + 6xy)y' &= -3y^2 \\ y' &= \frac{-3y^2}{3y^2 + 6xy}\end{aligned}$$

Note that this formula for y' involves both x and y . Implicit differentiation can be very useful for taking the derivatives of inverse functions.

For instance,

$$y = \sin^{-1} x \Rightarrow \sin y = x$$

Implicit differentiation yields

$$(\cos y)y' = 1$$

and

$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

Specific differentiation formulas

You will be responsible for knowing formulas for the derivatives and how to deduce these formulas from previous information: x^n , $\sin^{-1} x$, $\tan^{-1} x$, $\sin x$, $\cos x$, $\tan x$, $\sec x$, e^x , $\ln x$.

For example, let's calculate $\frac{d}{dx} \sec x$:

$$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} = \frac{-(-\sin x)}{\cos^2 x} = \tan x \sec x$$

You may be asked to find $\frac{d}{dx} \sin x$ or $\frac{d}{dx} \cos x$, using the following information:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\sin(h)}{h} &= 1 \\ \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} &= 0 \end{aligned}$$

Remember the definition of the derivative:

$$\frac{d}{dx} f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Tying up a loose end

How to find $\frac{d}{dx} x^r$, where r is a real (but not necessarily rational) number? All we have done so far is the case of rational numbers, using implicit differentiation. We can do this two ways:

1st method: base e

$$\begin{aligned} x &= e^{\ln x} \\ x^r &= (e^{\ln x})^r = e^{r \ln x} \\ \frac{d}{dx} x^r &= \frac{d}{dx} e^{r \ln x} = e^{r \ln x} \frac{d}{dx} (r \ln x) = e^{r \ln x} \frac{r}{x} \\ \frac{d}{dx} x^r &= x^r \left(\frac{r}{x} \right) = r x^{r-1} \end{aligned}$$

2nd method: logarithmic differentiation

$$\begin{aligned} (\ln f)' &= \frac{f'}{f} \\ f &= x^r \\ \ln f &= r \ln x \\ (\ln f)' &= \frac{r}{x} \\ f' = f(\ln f)' &= x^r \left(\frac{r}{x} \right) = r x^{r-1} \end{aligned}$$

Finally, in the first lecture I promised you that you'd learn to differentiate *anything*— even something as complicated as

$$\frac{d}{dx} e^{x \tan^{-1} x}$$

So let's do it!

$$\frac{d}{dx} e^{uv} = e^{uv} \frac{d}{dx} (uv) = e^{uv} (u'v + uv')$$

Substituting,

$$\frac{d}{dx} e^{x \tan^{-1} x} = e^{x \tan^{-1} x} \left(\tan^{-1} x + x \left(\frac{1}{1+x^2} \right) \right)$$